COM 6854: Verification and Testing

Exercise Sheet 2

Exercise 1: Analyse the Kangaroo-Puzzle using sets.

Exercise 2: Give the subset relation of the powerset of $\{1, 2, 3\}$ by drawing a graph with sets as nodes such that

- subsets are always below supersets and
- two sets are connected by an edge if one is a direct subset of the other.

A set B is a *direct subset* of a set A if $B \subseteq A$, $B \neq A$ and all other subsets C of A that are supersets of B must either be equal to A or to B.

Exercise 3: Show that the product of binary relations is associative:

$$R \circ (S \circ T) = (R \circ S) \circ T,$$

but not necessarily commutative:

$$R \circ S \neq S \circ R.$$

Exercise 4: Give examples of the following relations:

(a) irreflexive, i.e., $R \subseteq \overline{1_A}$,

(b) reflexive and transitive, but not symmetric,

(c) antisymmetric, but not transitive.

Exercise 5: In the kinship example from the lecture notes, prove that

(a) $mother = father \circ wife$,

(b) $mother \circ father^\circ = \emptyset$.