Pushdown Automata and Context Free Grammars – Lecture 11

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Theorem

A language is context free iff some pushdown automaton recognises it

language G is context free $\Leftrightarrow$ language G is recognised by a PDA

Lemma

If a language is context free some PDA recognises it

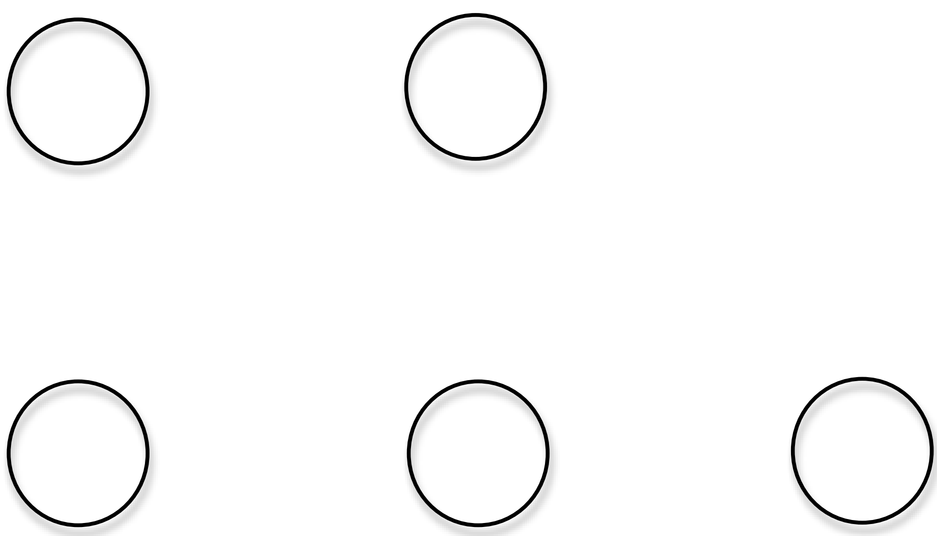
Proof Sketch (by Construction)

Design a PDA for a grammar that functions as follows:

```
Place '$' then start variable on stack
Repeat forever
     If stack top is variable symbol 'A'
          Select rule for 'A'
          Pop 'A'
          Push rhs of rule for 'A'
     While stack top is terminal symbol 'a'
          Read input and compare to 'a'
          If different
               Reject
     If stack top is '$'
          Accept
```

Example

$$A \rightarrow 0A1 \mid \varepsilon$$



Lemma

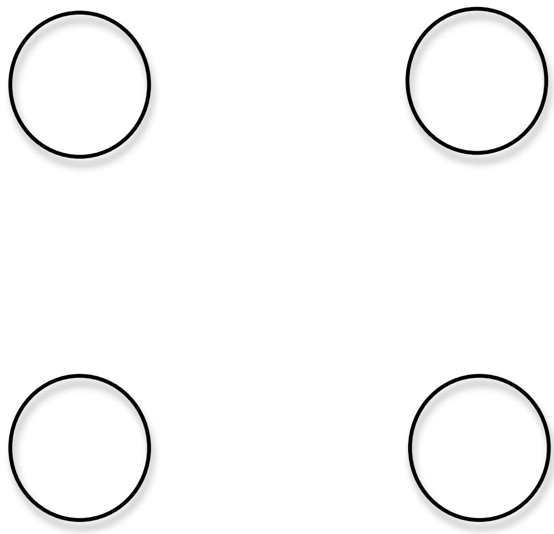
If a PDA recognises a language, then it is context free

Proof Sketch (by Construction)

For PDA $P = (Q,\Sigma,\Gamma,\delta,q_0,\{q_{accept}\})$ construct G with variables $\{A_{pq} \mid p,q \in Q\}$, start variable $A_{q_0,q_{accept}}$ and rules:

- $A_{pq} \to aA_{rs}b$
  for each $p,q,r,s \in Q$, $t \in \Gamma$ and $a,b \in \Sigma_\varepsilon$, if $\delta(p,a,\varepsilon)$ contains (r,t) and $\delta(s,b,t)$ contains (q,ε)
- $A_{pq} \to A_{pr}A_{rq}$
  for each $p,q,r \in Q$
- $A_{pp} \to \varepsilon$
  for each $p \in Q$

Example



$A_{q_0,q_3} \to$

$\delta(\ ,\ ,\varepsilon) = \{(\ ,\ ),\ldots\}$

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Claim (proof in Sipser)

A_{pq} generates string $x \Leftrightarrow x$ can bring PDA from state p with empty stack to q with empty stack