Hybrid Categorial Logics

MARK HEPPLE, *Department of Computer Science, University of Sheffield, Regents Court, Portobello Street, Sheffield, UK.*
E-mail: hepple@dcs.sheffield.ac.uk

Abstract

Recent work within Categorial Grammar has seen the development of a number of *multimodal* systems, where different families of connectives coexist within a single categorial logic. Such systems can be viewed as making available differing modes of linguistic description within a single grammatical formalism. This paper addresses proposals for constructing multimodal systems due to Hepple [7] and Moortgat & Oehrle [15], which are in many ways similar, but which make apparently contradictory claims concerning the appropriate interrelation of different modes of description, which lead in turn to differences for the kind of linguistic accounts that the two approaches make possible. Although we focus mostly on the view taken in Hepple [7], and its inspiration by earlier work involving structural modalities, the paper proceeds to a discussion of whether the two approaches are genuinely incompatible in the way that they at first appear.

1 Introduction

Categorial formalisms consist of logics, and different categorial formalisms use logics that differ in their limitations on the use of 'linguistic resources' in deduction, and their consequent sensitivity to the specific structuring of those resources. Comparison of logics in such terms (i.e. their resource usage characteristics) gives rise to the 'substructural hierarchy' of logics. Most categorial work has involved systems which are set at a single level of the substructural hierarchy, i.e. have just a single mode of resource usage. Such systems have turned out to be of restricted value for linguistic analysis, because of the complexities of resource usage to be found within any individual language, and, more generally, the differences of resource usage found between languages. Such limitations suggest the need for systems that allow exploitation of the resource usage characteristics of more than one substructural level.

One approach to realising this goal has employed *structural modalities*, which are unary operators that allow controlled access to the (more liberal) resource usage characteristics of higher substructural levels. In constructing a linguistic system under this approach, some specific resource logic must first be chosen as 'basic' for stating the grammar, thereby setting the *default* characteristics of resource sensitivity. Then, structural modalities are used to allow controlled access to different modes of resource usage. Various problems – theoretical, computational and practical – arise for the use of such operators. One practical consideration is that the need to have a single 'base' logic, which sets the default resource characteristics, presents problems for the development of a truly general cross-linguistic framework that is applicable to highly dissimilar languages. Furthermore, the complexity of syntactic analyses that require extensive use of structural modalities tends to encourage the selection of base logics that are stronger than might otherwise be chosen, with a concomitant loss of potentially useful resource sensitivity.

More recent work has seen the proposal of approaches which combine together substructural logics into a single *multimodal* system, i.e. where different families of connectives coexist within a single combined logic. Sublogics combined in this way may be familiar categorial systems (e.g. associative plus non-associative Lambek calculus), or may include families of connectives that are tailored to specific linguistic purposes, e.g. discontinuity phenomena.[1]

In this paper, I will be predominantly concerned with the proposals of Hepple [7] and Moortgat & Oehrle [15], who describe general approaches for combining different substructural subsystems into multimodal systems which allow type changes that exhibit 'movement between levels' (i.e. where an operator of one substructural level may be 'rewritten' to a corresponding one from another level). Although developed independently, the two sources propose similar methods for formulating multimodal logics. However, they make very different claims as to what constitute the appropriate relations between substructural levels, reflecting different intuitions as to the meanings of such sublogics within an overall multimodal system. This 'difference of opinion' between the two sources in turn leads to some differences for how multimodal systems may be used as linguistic formalisms, and for the kind of linguistic accounts they make possible.

In what follows, I will present the view of multimodal linkage taken in Hepple [7] – what I will call, for the convenience of having a name, the 'hybrid view'. This view is inspired by the earlier work using structural modalities, and its goal is to eliminate the need for structural modalities, whilst maintaining the descriptive power they provide. After introducing structural modalities, and the view of linkage that they inspire, I will go on to discuss some issues that arise for the use of a hybrid-style system as a linguistic formalism, and also the 'general linguistic approach' that the hybrid view tends to foster. Finally, I will directly address the question of which, if either, view of how linkages should be arranged is correct.

2 Categorial logics and substructural hierarchy

Categorial logics typically provide at least three connectives: a 'product' connective (corresponding to some linguistic structure-building operation), plus two implicational connectives (the product's left and right 'residuals') notated as $\stackrel{\circ}{\rightarrow}$ and $\stackrel{\circ}{\leftarrow}$ for a product $\circ$. Such a group of connectives minimally requires the sequent rules (2.1) and (2.2).[2]

The inclusion or otherwise of *structural rules* such as those in (2.3) will determine the characteristics of resource usage that the resulting system displays. With no structural rules, we have a version of the non-associative Lambek calculus (**NL**: [11]), where deduction is sensitive to the order and bracketing of assumptions, each of which must be used precisely once in a deduction. Adding [A] undermines sensitivity to specific bracketing, giving a version of the *associative* Lambek calculus (**L**: [10]). If [P] is also added, we have the system **LP** [2], which is closely akin to a fragment of linear logic [6]. The remaining logical possibility, having Permutation, but not association, is denoted **NLP**. For convenience, I will adopt distinct notations for the connectives of three of these systems, as follows: **L**:$\{\bullet,\backslash,/\}$, **NL**:$\{\odot,\oslash\!\!\!\backslash,\oslash\}$, **LP**:$\{\otimes,\multimap,\circ\!\!-\}$.

[1] Such discontinuity connectives are proposed in [12]. Regarding the formalisation and use of multimodal systems including such connectives, see [8], [9], [13], [18], [19].

[2] A *sequent* $\Gamma \Rightarrow A$ indicates that the *succedent* formula A can be derived from the *structured configuration* (i.e. non-empty bracketed sequence) of antecedent formulas Γ, where $(\cdot,\cdot)^{\circ}$ is the *structural connective* corresponding to the connective $\circ$. $\Gamma[\Phi']$ represents the result of replacing the subconfiguration Φ with Φ' in $\Gamma[\Phi]$.

(2.1) $A \Rightarrow A$ (id) $\qquad \dfrac{\Phi \Rightarrow B \quad \Gamma[B] \Rightarrow A}{\Gamma[\Phi] \Rightarrow A}[\text{cut}]$

(2.2) $\dfrac{(B, \Gamma)^\circ \Rightarrow A}{\Gamma \Rightarrow B\overset{\circ}{\to}A}[\overset{\circ}{\to}R] \qquad \dfrac{\Phi \Rightarrow C \quad \Gamma[B] \Rightarrow A}{\Gamma[(\Phi, C\overset{\circ}{\to}B)^\circ] \Rightarrow A}[\overset{\circ}{\to}L]$

$\dfrac{(\Gamma, B)^\circ \Rightarrow A}{\Gamma \Rightarrow A\overset{\circ}{\leftarrow}B}[\overset{\circ}{\leftarrow}R] \qquad \dfrac{\Phi \Rightarrow C \quad \Gamma[B] \Rightarrow A}{\Gamma[(B\overset{\circ}{\leftarrow}C, \Phi)^\circ] \Rightarrow A}[\overset{\circ}{\leftarrow}L]$

$\dfrac{\Gamma \Rightarrow A \quad \Phi \Rightarrow B}{(\Gamma, \Phi)^\circ \Rightarrow A{\circ}B}[\circ R] \qquad \dfrac{\Gamma[(B, C)^\circ] \Rightarrow A}{\Gamma[B{\circ}C] \Rightarrow A}[\circ L]$

(2.3) $\dfrac{\Gamma[(B, (C, D)^\circ)^\circ] \Rightarrow A}{\Gamma[((B, C)^\circ, D)^\circ] \Rightarrow A}[A] \qquad \dfrac{\Gamma[(B, C)^\circ] \Rightarrow A}{\Gamma[(C, B)^\circ] \Rightarrow A}[P]$

There are further possible structural rules, notably Weakening and Contraction, which allow, respectively, that any resource may be ignored or may be multiply used. These rules, together with those above, allow yet further logics. Comparison of logics in terms of the freedom they allow in resource usage, where additional structural rules means greater resource freedom, gives rise to the 'substructural hierarchy of logics'.

3 Multimodal logics

The possibility arises to combine more than one substructural logic into a mixed or *multimodal* system. For example, we might have a system which includes connectives for more than one of the above systems, and allow the logical rules (2.2) to operate schematically with respect to them. Schematic structural rules, conditioned to apply in only appropriate circumstances, would also be required, as (e.g.) in (3.1).

(3.1) $\dfrac{\Gamma[(B, (C, D)^\circ)^\circ] \Rightarrow A}{\Gamma[((B, C)^\circ, D)^\circ] \Rightarrow A}[A] \ \Big\{ \circ \in \{\bullet, \otimes\} \qquad \dfrac{\Gamma[(B, C)^\circ] \Rightarrow A}{\Gamma[(C, B)^\circ] \Rightarrow A}[P] \ \Big\{ \circ \in \{\odot, \otimes\}$

As things stand, the different levels within such a system would *coexist*, but would not, in any interesting way, be *interrelated*. In particular, the logic displays, in its derivability behaviour, no interesting relationships between the connectives of different levels, i.e. relationships revealing connections between the 'meaning' of these operators, as might be shown by a transition such as (e.g.) $A{\circ_i}B \Rightarrow A{\circ_j}B$. Such 'linkage' between levels can be effected by including a 'quasi-structural rule' such as (3.2), which makes possible the derivation (3.3) of $A{\circ_i}B \Rightarrow A{\circ_j}B$. It is interesting to observe that the same linkage rule yields the converse direction of rewriting between the implicational connectives of the two subsystems, i.e. alongside $A{\circ_i}B \Rightarrow A{\circ_j}B$, we have also $A\overset{\circ_j}{\leftarrow}B \Rightarrow A\overset{\circ_i}{\leftarrow}B$, as derived in (3.4).

The rule (3.2) is what Moortgat & Oehrle [15] term an 'inclusion' rule. A further possibility for rules relating different substructural levels is what Moortgat &

Oehrle [15] call 'interaction' rules, which are structural rules in which the structural connectives (i.e. antecedent brackets) of more than one level appear, as (e.g.) in (3.5).

$$(3.2) \qquad \frac{\Gamma[(\mathrm{B},\ \mathrm{C})^{\circ_j}] \Rightarrow \mathrm{A}}{\Gamma[(\mathrm{B},\ \mathrm{C})^{\circ_i}] \Rightarrow \mathrm{A}}[<]$$

$$(3.3) \qquad \cfrac{\cfrac{\cfrac{\mathrm{B} \Rightarrow \mathrm{B} \qquad \mathrm{A} \Rightarrow \mathrm{A}}{(\mathrm{A},\ \mathrm{B})^{\circ_j} \Rightarrow \mathrm{A}\circ_j\mathrm{B}}[\circ_j\mathrm{R}]}{(\mathrm{A},\ \mathrm{B})^{\circ_i} \Rightarrow \mathrm{A}\circ_j\mathrm{B}}[<]}{\mathrm{A}\circ_i\mathrm{B} \Rightarrow \mathrm{A}\circ_j\mathrm{B}}[\circ_i\mathrm{L}]$$

$$(3.4) \qquad \cfrac{\cfrac{\cfrac{\mathrm{A} \Rightarrow \mathrm{A} \qquad \mathrm{B} \Rightarrow \mathrm{B}}{(\mathrm{A}\overset{\circ_j}{\leftarrow}\mathrm{B},\ \mathrm{B})^{\circ_j} \Rightarrow \mathrm{A}}[\overset{\circ_j}{\leftarrow}\mathrm{L}]}{(\mathrm{A}\overset{\circ_j}{\leftarrow}\mathrm{B},\ \mathrm{B})^{\circ_i} \Rightarrow \mathrm{A}}[<]}{\mathrm{A}\overset{\circ_j}{\leftarrow}\mathrm{B} \Rightarrow \mathrm{A}\overset{\circ_i}{\leftarrow}\mathrm{B}}[\overset{\circ_i}{\leftarrow}\mathrm{R}]$$

$$(3.5) \qquad \frac{\Gamma[(\mathrm{B},\ (\mathrm{C},\ \mathrm{D})^{\circ_i})^{\circ_j}] \Rightarrow \mathrm{A}}{\Gamma[((\mathrm{B},\ \mathrm{C})^{\circ_j},\ \mathrm{D})^{\circ_i}] \Rightarrow \mathrm{A}} \text{ (double line)}$$

We have now seen all the components that are required for stating a multimodal logic. Approaches to constructing multimodal or 'hybrid' categorial logics along the lines just sketched have been been independently developed and proposed by Hepple [7] and Moortgat & Oehrle [15, 16]. Although these two sources disagree systematically on some matters concerning the relationships between levels, the details of how they formulate such mixed systems (or more precisely, provide sequent formulations) differ in only relatively minor regards.

Multimodal logics can be given an algebraic semantics whose components link quite intuitively with the components of the logic. Moortgat & Oehrle [16] adapt a method used with unimodal categorial logics, employing ternary Kripke-style relational structures. In a frame $\langle W, R^3\rangle$, the ternary relation R may be seen as corresponding to some mode of linguistic composition (where z in $Rzxy$ arises by appropriate composition of x and y) – the mode of composition which, intuitively, underlies the logic's product and residuals. In the multimodal case, there are multiple such relations (one for each group of connectives). A relation between levels in a mixed logic, as revealed by a theorem such as $\mathrm{A}\circ_i\mathrm{B} \Rightarrow \mathrm{A}\circ_j\mathrm{B}$, corresponds in the semantics to a linkage between the relevant relations R_i, R_j, such as $(\forall x, y, z \in W).R_izxy \rightarrow R_jzxy$.[3]

Despite the considerable similarity of the suggestions of [7] and [15] on *how* a multimodal system may be formulated, they disagree quite strikingly on *what* transitions between levels should be allowed. In particular, the two approaches take systematically contrary positions on the 'direction of movement' between levels. The view taken by Hepple [7] – what I have called the 'hybrid view' – is inspired by ideas involving *structural modalities* and *embedding translations*, which I will present next.

4 Structural modalities and embedding translations

Structural modalities are unary modal operators that may be used to allow controlled involvement of structural rules which are otherwise unavailable in a system, and thereby controlled access to the resource usage characteristics of stronger logics than that which, in the given case, is being used. The original structural modalities are the 'exponentials' ! and ? of linear logic [6], which give controlled reintroduction of the structural rules Contraction and Weakening (which account for the resource usage

[3] Hence the use of the term 'inclusion'. Hepple [7] provides a semantics for multimodal logics that is adapted from the less general groupoid approach to interpreting categorial logics.

differences between linear logic and intuitionistic logic). Morrill *et al.* [17] (see also [1]) suggest a number of structural modalities having possible linguistic uses.

We shall consider a simple example of a structural modality, and its use to give controlled involvement of a structural rule. In a sequent formulation, a structural modality will be associated with one or more modified structural rules which differ from their more general counterparts in requiring that one or more of the formulas directly affected by the rule's use are marked with a given modality. For example, in a system where permutation is not freely available, *restricted* involvement of [P] might be allowed via a unary operator $\triangle$, having the sequent rules (4.1) (where $\triangle\Gamma$ indicates a configuration in which all types have the form $\triangle$X):

(4.1) $$\frac{\Gamma[(\triangle B,\ C)^\circ] \Rightarrow A}{\Gamma[(C,\ \triangle B)^\circ] \Rightarrow A}[\triangle P] \qquad \frac{\triangle\Gamma \Rightarrow A}{\triangle\Gamma \Rightarrow \triangle A}[\triangle R] \qquad \frac{\Gamma[B] \Rightarrow A}{\Gamma[\triangle B] \Rightarrow A}[\triangle L]$$

The restricted permutation rule [$\triangle$P] allows any formula of the form $\triangle$X to permute freely, i.e. undermining linear order for just this assumption. The other rules are as for necessity in the modal logic S4. The left rule [$\triangle$L] plays a key role, allowing a $\triangle$ modality to be discarded (e.g. we have the theorem $\triangle X \Rightarrow X$), so that a permutable assumption can be transformed to a non-modal one having a fixed linear position. Such permutative modalities have been used in treatments of extraction, in particular being needed to allow for cases where moved elements do not originate in peripheral position within the domains from which they are extracted. Other such modalities can be used to give controlled reintroduction of other structural rules, e.g. an associativity modality could be used within a non-associative system, and so on.

Where a weaker logic is augmented with an appropriate structural modality, the resulting system is in general at least as strong as the relevant stronger logic (i.e. that stronger logic which differs from the weaker one only in its free availability of the structural rule that is associated with the structural modality). In particular, it is possible to recreate the derivability characteristics of the stronger logic within the weaker one by the use of a 'global modalisation strategy' that has the effect of making the relevant structural rule available wherever needed for that derivation. Such a 'global modalisation strategy' is stated as an *embedding translation.* For example, the translation (4.2) embeds a fragment of **LP** within **L**, so that $\Gamma \Rightarrow A$ is a theorem of the former *iff* $\triangle|\Gamma| \Rightarrow |A|$ is a theorem of the latter (see [5]). Another example is that intuitionistic logic can be embedded within (intuitionistic) linear logic, via an embedding translation using the exponentials ! and ?.

(4.2)

$\|A\|$	:=	A	(*where* A *is atomic*)
$\|A\otimes B\|$	:=	$(\triangle\|A\|) \bullet (\triangle\|B\|)$	
$\|B\multimap A)\|$	:=	$(\triangle\|B\|) \setminus \|A\|$	
$\|A\multimapinv B)\|$	:=	$\|A\| / (\triangle\|B\|)$	

5 Relations between substructural levels

Consider again the embedding translation (4.2). The embedding shows that **LP** can be 'represented' within the system '**L** *plus* permutation modality' ('**L**$\triangle$'). Of course, it is trivially true that **L** can also be 'represented' within **L**$\triangle$, and so **L**$\triangle$ provides a realm in which we can, in a sense, observe coexistence of **L** and **LP** (or at least of 'images' of these systems), and observe how the two systems interrelate.

Consider, for example, the **LP** formula $\mathrm{X}\otimes\mathrm{Y}$ and its translation $(\triangle\mathrm{X})\bullet(\triangle\mathrm{Y})$ under (4.2) (or strictly its translation assuming X,Y are atomic). The modalities indicate that the X,Y subformulas may appear in either order, i.e. we observe the interderivability $(\triangle\mathrm{X})\bullet(\triangle\mathrm{Y})\Leftrightarrow(\triangle\mathrm{Y})\bullet(\triangle\mathrm{X})$, akin to $\mathrm{X}\otimes\mathrm{Y} \Leftrightarrow \mathrm{Y}\otimes\mathrm{X}$ for the original formula. The $\triangle$ modalities may be 'discarded', e.g. we have $(\triangle\mathrm{X})\bullet(\triangle\mathrm{Y}) \Rightarrow \mathrm{X}\bullet\mathrm{Y}$ (and also $(\triangle\mathrm{X})\bullet(\triangle\mathrm{Y}) \Rightarrow \mathrm{Y}\bullet\mathrm{X}$), a step corresponding to selection of, or commitment to, one of the two permitted orders. This latter theorem suggests how $\otimes$ and $\bullet$ might be related in a multimodal logic, i.e. so that $\mathrm{X}\otimes\mathrm{Y} \Rightarrow \mathrm{X}\bullet\mathrm{Y}$ is a theorem, as if $\mathrm{X}\otimes\mathrm{Y}$ is in some sense 'implicitly modalised' relative to $\mathrm{X}\bullet\mathrm{Y}$.

Consider next the **LP** implicational $\mathrm{X}\circ\!-\mathrm{Y}$, and its translation $\mathrm{X}/(\triangle\mathrm{Y})$. This formula exhibits the interderivability $\mathrm{X}/(\triangle\mathrm{Y}) \Leftrightarrow (\triangle\mathrm{Y})\backslash\mathrm{X}$, akin to $\mathrm{X}\circ\!-\mathrm{Y} \Leftrightarrow \mathrm{Y}-\!\circ\mathrm{X}$. Note that **L**$\triangle$ allows the transition $\mathrm{X}/\mathrm{Y} \Rightarrow \mathrm{X}/(\triangle\mathrm{Y})$, suggesting $\mathrm{X}/\mathrm{Y} \Rightarrow \mathrm{X}\circ\!-\mathrm{Y}$ as a theorem of a mixed logic where there is coexistence of these two levels.

The above line of argument, applied to other substructural levels and modalities, suggests a general view of linkage such that for any two products $\circ_i$ and $\circ_j$, where the former has greater freedom for resource usage than the latter (i.e. where the former's structural rules are a superset of the latter's), then we can expect characteristic theorems for the mixed logic such as:

$$\mathrm{A}\circ_i\mathrm{B} \Rightarrow \mathrm{A}\circ_j\mathrm{B}$$
$$\mathrm{A}\overset{\circ_j}{\leftarrow}\mathrm{B} \Rightarrow \mathrm{A}\overset{\circ_i}{\leftarrow}\mathrm{B}$$

6 Term assignment

I will next describe a system of term assignment for the hybrid approach, under which the formulas in any proof are associated with lambda terms in accordance with the well known Curry–Howard interpretation (Howard 1969). The term associated with any succedent formula records the *natural deduction* structure of the dominating subproof. These terms play a role in handling NL semantics and word order, as we shall see shortly.

The sequent rules are restated below with term labelling. Antecedent formulas are associated with variables. Cut inferences are interpreted via substitution (where $a[b/v]$ represents the substitution of b for v in a). For implicational connectives, left and right inferences are interpreted via functional application and abstraction, respectively. Note that a different abstraction and application operator is required for each implicational, so that terms fully record the proof structure.[4] The implication $\overset{\circ}{\leftarrow}$ (resp. $\overset{\circ}{\rightarrow}$) has application operator $\underset{\circ}{\leftarrow}$ (resp. $\underset{\circ}{\rightarrow}$), giving $a\underset{\circ}{\leftarrow}b$ (resp. $b\underset{\circ}{\rightarrow}a$) for '$a$ applied to b', and abstraction operator $[\underset{\circ}{\leftarrow}]$ (resp. $[\underset{\circ}{\rightarrow}]$), e.g. $[\underset{\circ}{\leftarrow}]v.a$ (resp. $[\underset{\circ}{\rightarrow}]v.a$) for abstraction over v in a. Product right inferences are interpreted via system specific pairing, and for product left inferences, a term such as $[x/v\circ w].a$ implicitly represents the substitution of x for $v{+}w$ in a.[5] A labelled version of the (schematic) inclusion

[4] See Buszkowski [4] and Wansing [20] for augmented term systems implementing the 'formulas-as-types' notion for a variety of substructural logics.

[5] This operator is essentially a compact notation for one used by Benton *et al.* (1992) with linear logic.

rule [<] is shown in (6.3). Note that the rule is 'neutral' regarding term assignment, i.e. all premise sequent labels are passed on unchanged. Any structural rules will be similarly neutral (including 'interaction' cases). The proof (6.4) illustrates this system of term assignment.

(6.1)

$$\mathrm{A}:v \Rightarrow \mathrm{A}:v \quad \text{(id)} \qquad\qquad \dfrac{\Phi \Rightarrow \mathrm{B}:b \qquad \Gamma[\mathrm{B}:v] \Rightarrow \mathrm{A}:a}{\Gamma[\Phi] \Rightarrow \mathrm{A}:a[b/v]}\,[\mathrm{cut}]$$

(6.2)

$$\dfrac{(\mathrm{B}:v,\ \Gamma)^{\circ} \Rightarrow \mathrm{A}:a}{\Gamma \Rightarrow \mathrm{B}\overset{\circ}{\rightarrow}\mathrm{A}:[\underset{\circ}{\rightarrow}]v.a}\,[\overset{\circ}{\rightarrow}\mathrm{R}] \qquad \dfrac{\Phi \Rightarrow \mathrm{C}:c \qquad \Gamma[\mathrm{B}:v] \Rightarrow \mathrm{A}:a}{\Gamma[(\Phi,\ \mathrm{C}\overset{\circ}{\rightarrow}\mathrm{B}:w)^{\circ}] \Rightarrow \mathrm{A}:a[(c \underset{\circ}{\rightarrow} w)/v]}\,[\overset{\circ}{\rightarrow}\mathrm{L}]$$

$$\dfrac{(\Gamma,\ \mathrm{B}:v)^{\circ} \Rightarrow \mathrm{A}:a}{\Gamma \Rightarrow \mathrm{A}\overset{\circ}{\leftarrow}\mathrm{B}:[\underset{\circ}{\leftarrow}]v.a}\,[\overset{\circ}{\leftarrow}\mathrm{R}] \qquad \dfrac{\Phi \Rightarrow \mathrm{C}:c \qquad \Gamma[\mathrm{B}:v] \Rightarrow \mathrm{A}:a}{\Gamma[(\mathrm{B}\overset{\circ}{\leftarrow}\mathrm{C}:w,\ \Phi)^{\circ}] \Rightarrow \mathrm{A}:a[(w \underset{\circ}{\leftarrow} c)/v]}\,[\overset{\circ}{\leftarrow}\mathrm{L}]$$

$$\dfrac{\Gamma \Rightarrow \mathrm{A}:a \qquad \Phi \Rightarrow \mathrm{B}:b}{(\Gamma,\ \Phi)^{\circ} \Rightarrow \mathrm{A}\circ\mathrm{B}:\langle a,b\rangle^{\circ}}\,[\circ\mathrm{R}] \qquad \dfrac{\Gamma[(\mathrm{B}:v,\ \mathrm{C}:w)^{\circ}] \Rightarrow \mathrm{A}:a}{\Gamma[\mathrm{B}\circ\mathrm{C}:x] \Rightarrow \mathrm{A}:[x/v\circ w].a}\,[\circ\mathrm{L}]$$

(6.3)

$$\dfrac{\Gamma[(\mathrm{B}:b,\ \mathrm{C}:c)^{\circ_j}] \Rightarrow \mathrm{A}:a}{\Gamma[(\mathrm{B}:b,\ \mathrm{C}:c)^{\circ_i}] \Rightarrow \mathrm{A}:a}\,[<]$$

(6.4)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{C}:z \Rightarrow \mathrm{C}:z \qquad \mathrm{B}:w \Rightarrow \mathrm{B}:w}{(\mathrm{C}:z,\ \mathrm{C}\obslash\mathrm{B}:y)^{\odot} \Rightarrow \mathrm{B}:z \underset{\odot}{\rightarrow} y}\,[\obslash\mathrm{L}] \qquad \mathrm{A}:v \Rightarrow \mathrm{A}:v}{(\mathrm{A/B}:x,\ (\mathrm{C}:z,\ \mathrm{C}\obslash\mathrm{B}:y)^{\odot}\ )^{\bullet} \Rightarrow \mathrm{A}:x \underset{\bullet}{\leftarrow} (z \underset{\odot}{\rightarrow} y)}\,[/\mathrm{L}]}{(\mathrm{A/B}:x,\ (\mathrm{C}:z,\ \mathrm{C}\obslash\mathrm{B}:y)^{\otimes}\ )^{\bullet} \Rightarrow \mathrm{A}:x \underset{\bullet}{\leftarrow} (z \underset{\odot}{\rightarrow} y)}\,[<]}{(\mathrm{A/B}:x,\ (\mathrm{C}:z,\ \mathrm{C}\obslash\mathrm{B}:y)^{\otimes}\ )^{\otimes} \Rightarrow \mathrm{A}:x \underset{\bullet}{\leftarrow} (z \underset{\odot}{\rightarrow} y)}\,[<]}{(\mathrm{A/B}:x,\ (\mathrm{C}\obslash\mathrm{B}:y,\ \mathrm{C}:z)^{\otimes}\ )^{\otimes} \Rightarrow \mathrm{A}:x \underset{\bullet}{\leftarrow} (z \underset{\odot}{\rightarrow} y)}\,[\mathrm{P}]}{((\mathrm{A/B}:x,\ \mathrm{C}\obslash\mathrm{B}:y)^{\otimes},\ \mathrm{C}:z)^{\otimes} \Rightarrow \mathrm{A}:x \underset{\bullet}{\leftarrow} (z \underset{\odot}{\rightarrow} y)}\,[\mathrm{A}]}{(\mathrm{A/B}:x,\ \mathrm{C}\obslash\mathrm{B}:y)^{\otimes} \Rightarrow \mathrm{A}\circ\!\!-\mathrm{C}:[\underset{\otimes}{\leftarrow}]z.x \underset{\bullet}{\leftarrow} (z \underset{\odot}{\rightarrow} y)}\,[\circ\!\!-\mathrm{R}]$$

7 Word order and semantics in a hybrid formalism

It remains to be shown that a system which has (or at least includes) the hybrid direction of linkage between levels, i.e. the direction suggested by structural modalities, can usefully be employed as a linguistic formalism. Although the hybrid direction of linkage, as compared with its converse, requires no special features in the formulation of the logic, it does have some consequences for how such a system may be used in linguistic analysis, specifically in regard to the treatment of word order. Consider, for example a system which includes **L** and **LP** as levels, and a theorem $\Gamma \Rightarrow \mathrm{A}$, corresponding to some linguistic combination, which can be derived just within **L**. Such a mixed system will also admit alternative theorems $\Gamma' \Rightarrow \mathrm{A}$, where Γ' is some configuration of precisely the formulas in Γ under *any alternative ordering* thereof.[6]

One possible response to this problem might be to limit our attention, for linguistic purposes, to only those sequents whose antecedents are configured with structural connectives that *do* clearly order their subconfigurations, e.g. so that $(\cdot,\cdot)^{\bullet}$ may appear in the end-sequent of a linguistic derivation, but not $(\cdot,\cdot)^{\otimes}$, although the latter might appear elsewhere in the body of the proof.[7] Such a requirement would seem too strict, however, since it completely rules out the possibility of having lexical type assignments that do not strictly order functor and argument, a decision which should surely be in the hands of the grammar writer, rather than a fixed requirement of the formalism.

The solution to this problem pursued in Hepple [7] is based on the system of term assignment described above, i.e. with the word order consequences of proofs being derived from their proof terms. Recall that in the categorial approach, the word order requirements of lexical items is specified via the connectives of their lexical type assignment. Given the rich term algebra described above, proof terms fully record all such information. To extract the word order consequences of a proof, its proof term is first normalised,[8] reducing it to a form in which the linear order, etc, information originally encoded by its antecedent types is most directly expressed. This information can then be used in deriving an ordering over the free variables of the term, which in turn implies an ordering of the types combined.

Consider, for example, the term $[\overset{\leftarrow}{\otimes}]z.x\overset{\leftarrow}{\bullet}(z\overset{\rightarrow}{\odot}y)$ generated by (6.4). The directionalities of applications suggests the ordering $x \prec z \prec y$ over variables. Abstraction discounts z as an 'orderable element', leaving just $x \prec y$, i.e. with A/B preceding $\mathrm{C}\backslash_{\odot}\mathrm{B}$, as we would expect from the former's directionality. For a term $x\overset{\leftarrow}{\otimes}y$, the permutativity of $\otimes$ suggests that both orderings of x and y are possible. Note however that order determination must be sensitive to the specific modes of structuring and their properties, e.g. the non-associativity of $\odot$ implies an 'integrity' for y, z in $x\overset{\leftarrow}{\otimes}(y\overset{\leftarrow}{\odot}z)$ excluding $y \prec x \prec z$ as a possible order, despite the permutativity of $\otimes$.

The method for determining order from a normalised proof term (which I will simply sketch here) involves firstly transforming it to give a further term – its *yield* term – in which the original term's orderable elements are structured in accordance with their original manner of combination, using operators that I will notate identically to the corresponding type constructors, as in the following examples:

$$x\overset{\leftarrow}{\bullet}(z\overset{\rightarrow}{\bullet}y) \mapsto x\bullet(z\odot y)$$
$$[\overset{\leftarrow}{\otimes}]z.\,x\overset{\leftarrow}{\bullet}(z\overset{\rightarrow}{\odot}y) \mapsto x\bullet y$$
$$[(v\overset{\leftarrow}{\otimes}w)/x\bullet y].\langle x,y\rangle^{\odot} \mapsto (v\otimes w)$$

Yield terms may be restructured in ways appropriate to the different operators (e.g.

[6] Any proof of $\Gamma \Rightarrow$ A may be extended by multiple [<] inferences to give a proof of $\Phi \Rightarrow$ A, where Φ is just like Γ except all bracket pairs are $()^{\otimes}$. Extending this proof with repeated uses of [P] and [A], we can attain any desired reordering of the antecedent types.

[7] This idea is adapted from the Moortgat & Oehrle [16] treatment of head wrapping phenomena, where certain structural connectives are designated as 'abstract', meaning 'without phonetic interpretation', and hence are not allowed to appear in sequents corresponding to linguistic combinations.

[8] Such normalisation requires the following conversion rules:

$$([\overset{\leftarrow}{\circ}]v.a)\overset{\leftarrow}{\circ}b \overset{\beta}{\leadsto} a[b/v] \qquad ([b/v\circ' w].a)\overset{\leftarrow}{\circ}c \overset{c}{\leadsto} [b/v\circ' w].(a\overset{\leftarrow}{\circ}c)$$
$$b\overset{\rightarrow}{\circ}([\overset{\rightarrow}{\circ}]v.a) \overset{\beta}{\leadsto} a[b/v] \qquad c\overset{\rightarrow}{\circ}([b/v\circ' w].a) \overset{c}{\leadsto} [b/v\circ' w].(c\overset{\rightarrow}{\circ}a)$$
$$[\langle b,c\rangle^{\circ}/v\circ w].a \overset{\beta}{\leadsto} a[b/v,c/w]$$

subterms $p \otimes q$ may be rewritten to $q \otimes p$, etc.). Possible linear orders can simply be 'read off' the variants a yield term under restructuring, e.g. $x \overset{\leftarrow}{\otimes} (y \overset{\leftarrow}{\odot} z)$ gives orders xyz and yzx, since its yield term is $x \otimes (y \odot z)$, whose only variant is $(y \odot z) \otimes x$

The lambda terms produced by proofs are also useful for handling the natural language semantic consequences of type combinations. The fine-grained distinctions they encode, between different modes of construction, are inappropriate for linguistic semantics, but such terms can easily be transformed to simpler ones, employing only a single form of abstraction (λ) and with application notated by left-right juxtaposition.

8 The hybrid linguistic model

I noted earlier that one problem of a structural modality based approach is that it tends to favour the selection of a relatively strong system for the base logic, i.e. because the weaker the base level logic, the more extensive will be the need to use structural modalities, giving rather complicated analyses. The choice of a stronger base logic is associated with a loss of potentially useful resource sensitivity. This problem does not arise for the hybrid approach, which freely allows us to use weaker logics in specifying lexical types that richly encode linguistic information.

For example, consider a multimodal system that includes only the levels **L** and **LP**. Of these two levels, **L** is clearly the one that will in general be more appropriate for linguistic description. Under the hybrid view, the linkage between these two levels is such that $X \otimes Y \Rightarrow X \bullet Y$ is a theorem, alongside which we will find also (e.g.) $X/Y \Rightarrow X \multimapinv Y$. Note that it is the latter theorem, and its variants, that most crucially bear upon what is gained by the move to a mixed system, given that the lexical encoding of linguistic information predominantly involves the assignment of *functional* types. Hence, a lexical functor constructed with **L** connectives may be transformed to one involving **LP** connectives, allowing us to exploit the structural freedom of that level. For example, the availability of the **LP** level, and its permutative character, allows for a possible treatment of extraction phenomena, whereby a 'sentence missing NP somewhere' may be derived as $\mathrm{s} \multimapinv \mathrm{np}$. This possible treatment is illustrated by the proof (8.1). If we take the proof term generated by this proof, and substitute for variables the corresponding word atoms, we would have a term: $who([\overset{\leftarrow}{\otimes}]v.(kim \overset{\rightarrow}{\bullet} ((sent \overset{\leftarrow}{\bullet} v) \overset{\leftarrow}{\bullet} away)))$, which gives a total ordering over word atoms: $who \prec kim \prec sent \prec away$ i.e. this proof constitutes a derivation for *who Kim sent away*. Note that with the converse direction of linkage between **L** and **LP**, but with lexical specification still exploiting the connectives of **L**, *no* practical use of the **LP** level would arise.

In practice, it is likely that a weaker logic would be preferred for lexical specification, as this would enable more information to be encoded in lexical types. For example, lexical encoding using (predominantly) the level **NL** would allow us to identify during syntactic derivation, what was lexically given argument order, i.e. since any 'non-associative functor', of the form $A \not{/} B$ or $B \not{\backslash} A$, would have to be a 'natural projection' of some lexical head. Given the hybrid pattern of 'movement between levels', such lexical specification is still compatible with the above approach to extraction (i.e. since we have, most importantly, transformations $X \not{/} Y \Rightarrow X/Y \Rightarrow X \multimapinv Y$). Even weaker logics would allow yet further information to be lexically encoded. For example, it is likely that lexical assignments should specify *headedness* or *dependency* information, as in the calculi of Moortgat & Morrill [14].

(8.1)

$$\dfrac{np \Rightarrow np \qquad s \Rightarrow s}{(np,\ np\backslash s)^{\bullet} \Rightarrow s}[\backslash L]$$

$$\dfrac{(np,\ np\backslash s)^{\bullet} \Rightarrow s \qquad pp \Rightarrow pp}{(np,\ ((np\backslash s)/pp,\ pp)^{\bullet})^{\bullet} \Rightarrow s}[/L]$$

$$\dfrac{(np,\ ((np\backslash s)/pp,\ pp)^{\bullet})^{\bullet} \Rightarrow s \qquad np \Rightarrow np}{(np,\ ((((np\backslash s)/pp)/np,\ np)^{\bullet},\ pp)^{\bullet})^{\bullet} \Rightarrow s}[/L]$$

$$\dfrac{(np,\ ((((np\backslash s)/pp)/np,\ np)^{\bullet},\ pp)^{\bullet})^{\bullet} \Rightarrow s}{(np,\ ((((np\backslash s)/pp)/np,\ np)^{\otimes},\ pp)^{\bullet})^{\bullet} \Rightarrow s}[<]$$

$$\dfrac{(np,\ ((((np\backslash s)/pp)/np,\ np)^{\otimes},\ pp)^{\bullet})^{\bullet} \Rightarrow s}{(np,\ ((((np\backslash s)/pp)/np,\ np)^{\otimes},\ pp)^{\otimes})^{\bullet} \Rightarrow s}[<]$$

$$\dfrac{(np,\ ((((np\backslash s)/pp)/np,\ np)^{\otimes},\ pp)^{\otimes})^{\bullet} \Rightarrow s}{(np,\ ((((np\backslash s)/pp)/np,\ np)^{\otimes},\ pp)^{\otimes})^{\otimes} \Rightarrow s}[<]$$

$$\dfrac{(np,\ ((((np\backslash s)/pp)/np,\ np)^{\otimes},\ pp)^{\otimes})^{\otimes} \Rightarrow s}{(np,\ (((np\backslash s)/pp)/np,\ (np,\ pp)^{\otimes})^{\otimes})^{\otimes} \Rightarrow s}[A]$$

$$\dfrac{(np,\ (((np\backslash s)/pp)/np,\ (np,\ pp)^{\otimes})^{\otimes})^{\otimes} \Rightarrow s}{(np,\ (((np\backslash s)/pp)/np,\ (pp,\ np)^{\otimes})^{\otimes})^{\otimes} \Rightarrow s}[P]$$

$$\dfrac{(np,\ (((np\backslash s)/pp)/np,\ (pp,\ np)^{\otimes})^{\otimes})^{\otimes} \Rightarrow s}{(np,\ ((((np\backslash s)/pp)/np,\ pp)^{\otimes},\ np)^{\otimes})^{\otimes} \Rightarrow s}[A]$$

$$\dfrac{(np,\ ((((np\backslash s)/pp)/np,\ pp)^{\otimes},\ np)^{\otimes})^{\otimes} \Rightarrow s}{((np,\ (((np\backslash s)/pp)/np,\ pp)^{\otimes})^{\otimes},\ np)^{\otimes} \Rightarrow s}[A]$$

$$\dfrac{((np,\ (((np\backslash s)/pp)/np,\ pp)^{\otimes})^{\otimes},\ np)^{\otimes} \Rightarrow s}{(np,\ (((np\backslash s)/pp)/np,\ pp)^{\otimes})^{\otimes} \Rightarrow s \circ\!\!- np}[\circ\!\!-R]$$

$$\dfrac{rel \Rightarrow rel \qquad (np,\ (((np\backslash s)/pp)/np,\ pp)^{\otimes})^{\otimes} \Rightarrow s \circ\!\!- np}{\underset{who}{(rel/(s \circ\!\!- np))},\ (\underset{Kim}{np},\ (\underset{sent}{((np\backslash s)/pp)/np},\ \underset{away}{pp})^{\otimes})^{\otimes})^{\bullet} \Rightarrow rel}[/L]$$

Hopefully, the above discussion will serve to give a feeling for the character of the linguistic model that the hybrid approach will favour, i.e. one with rich lexical encoding of syntactic information, achieved using predominantly the implicational connectives of the *weakest* sublogics, with the stronger sublogics of the mixed system allowing less informative descriptions of (functional) linguistic objects, of use in handling phenomena for which some lexically encoded information is not relevant and would get in the way of a simple and elegant account.

In closing this section, I will briefly mention a further elaboration that is possible for this formalism, and which will, I believe, greatly extend its power (in both formal and practical senses). This extension involves *lexical encoding of derivational structure*, in which lexical items are associated with complex string terms that are constructed using the operators of the proof term algebra, which (being isomorphic to proofs) can be seen to encode partial proofs. The proof structure that such lexical terms encode can be brought into play within linguistic derivations by appropriate use of normalisation, allowing further capabilities for lexical specification of linguistic information. See Hepple [9] for a detailed presentation of this method with a somewhat different multimodal system (which combines **L** with connectives for discontinuity).[9] Treatments of a range of phenomena are presented there which depend on the lexical encoding method, including, for example, treatments of quantification, pied-piping and gapping.[10]

---

[9] The lexical encoding method is also used in Hepple [7], but the presentation there is flawed by a failure to recognise that an appropriate involvement of normalisation is needed for all but the simplest of cases. For example, multiple quantifier cases cannot be derived without normalisation.

[10] Those accounts can be reconstructed within the present hybrid framework, using the **LP** connectives $\circ\!\!-, -\!\!\circ$ in place of the discontinuity connectives that are used there.

9 On two views of linkage

The view of linkage argued for in the previous section contradicts that taken by Moortgat & Oehrle [15, 16]. For example, they suggest $A \bullet B \Rightarrow A \otimes B$ as the characteristic theorem linking the **L** and **LP** levels. More generally, for a characteristic theorem $A \circ_i B \Rightarrow A \circ_j B$, the level of $\circ_i$ should have a subset of the structural rules of the level of $\circ_j$. This view is defended in terms of giving inter-level transformations that involve 'forgetting' or 'loss of information'. Thus, the more structural rules that a level has, the 'less informative' it is seen to be, since the less the structure that is preserved at that level. Since **LP** is permutative, the transformation $X \bullet Y \Rightarrow X \otimes Y$ is seen to involve the forgetting of order.

Despite its diametrically opposing idea of what constitute the 'natural linkages' between levels, the hybrid view can also be argued to allow only inter-level transformations that give 'loss of information'. According to this view, the permutativity $X \otimes Y \Leftrightarrow Y \otimes X$ is taken to indicate that *both* orders are possible, rather than that the ordering is unknown, and so the transformation $X \otimes Y \Rightarrow X \bullet Y$ can again be seen to involve 'loss of information', i.e. forgetting of one the two orders that are possible. In general for the hybrid view, the more structural rules a level has, the *more* informative it is seen to be, since the structural rules are seen to indicate a conjunction of alternative possibilities.

How are we to make sense of such contradictory views? Intuitively, when evaluated by their own criteria, both approaches appear to be correct in their claims of 'loss of information' in inter-level transitions. For Moortgat & Oehrle, the formula $X \otimes Y$ does not appear to order X and Y, since no order dependent use of its subtypes can be made. For the hybrid view, however, the formula $X \otimes Y$ *does* appear to encode *both* orders as being possible, precisely because it can be transformed to both $X \bullet Y$ and $Y \bullet X$. These comments suggest that the apparent disagreement may stem from the lack of a clear enough understanding of how to identify the 'meaning' of a level within a multimodal system. It may be that the meaning of a level within a multimodal system cannot be determined purely from level-internal considerations, because a vital component of this meaning is the level's linkages to other levels and its place within the overall system. In that case, it would be a mistake to expect a formula $X \otimes Y$ to have the same meaning in the alternative approaches, because the two **'LP'** levels, having different linkages within different multimodal systems, are consequently incomparable.

If this suggestion is correct, both Hepple [7] and Moortgat & Oehrle [15] are wrong to claim that the sublogics of a multimodal system *must* be ordered according to some criterion based on level-internal characteristics. Rather the very linking of levels determines their meaning, such that inter-level transitions must be well-behaved regarding 'loss of information' (even if only by a collapse to all levels having the same meaning, where a circular pattern of linkage is imposed). In that case, the possibility arises of a system having, amongst others, two distinct levels that *both* correspond to **LP** (when evaluated by purely internal criteria) but which play very different roles within the system due to their different linkages, i.e. with one level indicating conjunction, and the other underdetermination, of order.

10 Concluding remarks

I have described an approach for formulating multimodal logics in which 'movement between levels' is allowed in derivation, corresponding to what might be seen as movement between different modes of linguistic description. The pattern of movement between levels is inspired by consideration of categorial systems employing structural modalities, and multimodal systems so formulated should, I believe, allow reconstruction of many of the accounts that have depended on structural modalities, whilst allowing such operators to be dispensed with. The approach I have described is in many ways similar to that of Moortgat & Oehrle [15], the striking difference being diametrically opposing views of how the different sublogics should be linked. I have suggested that these two sources may be, in some sense, both right *and* both wrong: right in allowing the linkages they allow, and wrong in claiming that the alternative should be excluded. If this suggestion is correct, future multimodal systems may well exploit *both* directions of linkage. In that case, the special mechanisms required by the hybrid approach in relation to word order determination, or some alternative that fulfils the same role, will be required, so that the systems so developed should have much of the character of the approach described here.

References

[1] Barry, G., Hepple, M., Leslie, N. and Morrill, G. 1991. 'Proof figures and structural operators for categorial grammar'. *Proceedings of the 5th Conference of the European Chapter of the Association for Computational Linguistics (EACL–5)*, Berlin.

[2] van Benthem, J. 1983. 'The semantics of variety in Categorial Grammar.' Report 83-29, Department of Mathematics, Simon Fraser University. Also in W. Buszkowski, W. Marciszewski and J. van Benthem (eds), *Categorial Grammar*, Vol. 25, Linguistic and Literary Studies in Eastern Europe, John Benjamins, Amsterdam/Philadelphia. 1988.

[3] Benton, N., Bierman, G., de Paiva, V. & Hyland, M. 1992. 'Term assignment for intuitionistic linear logic.' Technical Report, Cambridge University Computer Laboratory.

[4] Buszkowski, W. 1987. 'The Logic of Types.' In J. Srzednicki (ed), *Initiatives in Logic*, Martinus Nijhoff Publishers, Dordrecht.

[5] Došen, K. 1990. 'Modal translations in substructural logics'. Report 10–90, Universitát Konstanz.

[6] Girard, J-Y. 1987. 'Linear Logic'. *Theoretical Computer Science* **59**, pp1-102.

[7] Hepple, M. 1993. 'A general framework for hybrid substructural categorial logics'. Ms. IRCS, UPenn. Available as IRCS Report 94–14.

[8] Hepple, M. 1994. 'Discontinuity and the Lambek Calculus'. *Proceedings of the 15th International Conference on Computational Linguistics (COLING'94)*.

[9] Hepple, M. 1994. 'Labelled deduction and discontinuous constituency'. M. Abrusci, C. Casadio and M. Moortgat (eds). *Linear Logic and Lambek Calculus*. Proceedings 1st Rome Workshop. Dynamic Interpretation of Natural Language, Esprit Basic Research Project 6852, Dyana Occasional Publications.

[10] Lambek, J. 1958. 'The mathematics of sentence structure'. *American Mathematical Monthly*, **65**, pp154–170.

[11] Lambek, J. 1961. 'On the calculus of syntactic types.' In R. Jakobson (ed), *Structure of Language and its Mathematical Aspects*, *Proceedings of the Symposia in Applied Mathematics XII*, American Mathematical Society.

[12] Moortgat, M. 1988. *Categorial Investigations: Logical and Linguistic Aspects of the Lambek Calculus*, Foris, Dordrecht.

[13] Moortgat, M. 1991. 'Generalized Quantification and Discontinuous Type Constructors'. To appear in H. Bunt & A. van Horck (eds) *Discontinuous Constituency*. De Gruyter, Berlin.

[14] Moortgat, M. & Morrill, G. 1991. 'Heads and Phrases: Type Calculus for Dependency and Constituency.' Ms OTS Utrecht. To appear: *Journal of Language, Logic and Information*.

[15] Moortgat, M. and Oehrle, R. 1993. *Logical parameters and linguistic variation. Lecture notes on categorial grammar.* Fifth European Summer School in Logic, Language and Information. Lisbon.

[16] Moortgat, M. and Oehrle, R. 1994. 'Adjacency, dependency and order'. *Proceedings of the 9th Amsterdam Colloquium.*

[17] Morrill, G., Leslie, N., Hepple, M. and Barry, G. 1990. 'Categorial deductions and structural operations.' In G. Barry and G. Morrill (eds), *Studies in Categorial Grammar.* Edinburgh Working Papers in Cognitive Science, Volume 5. Centre for Cognitive Science, University of Edinburgh.

[18] Morrill, G. 1993. 'Discontinuity and Pied-Piping in Categorial Grammar'. Research Report LSI–93–18–R. Departament de Llenguatges i Sistemes Informátics, Universitat Politécnica de Catalunya.

[19] Morrill, G. & Solias, M.T. 1993. 'Tuples, Discontinuity, and Gapping in Categorial Grammar.' *Proceedings of the 6th Conference of the European Chapter of the Association for Computational Linguistics (EACL–6)*, Utrecht.

[20] Wansing, W. 1990. 'Formulas-as-types for a hierarchy of sublogics of Intuitionistic Propositional Logic.' Ms. Institut für Philosophie, Freie Universität Berlin.

Received 7 January 1995