#### Personalized Health with Gaussian Processes

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Disease Mapping Workshop, Leahurst

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### Outline

#### Health

Data Heterogenity

Deep Learning

Conclusions

### What's Changed (Changing) for Medicine?

Modern data availability.



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Modern data availability.





#### Hensman et al. (2013)

#### Gaussian Processes for Big Data

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#### Abstract

We introduce stochastic variational inference for Gaussian process models. This enables the application of Gaussian process (GP) models to data sets containing millions of data points. We show how GPs can be variEven to accommodate these data sets, various approximate techniques are required. One approach is to partition the data set into separate groups [e.g. Snelson and Ghahramani, 2007, Urtasun and Darrell, 2008]. An alternative is to build a low rank approximation to the covariance matrix based around 'inducing variables' [see e.g. Csató and Opper, 2002, Seeger et al., 2003, Quiñonero Candela and Rasmussen, 2005, Ti-



#### Hensman et al. (2013)



Figure 4: Convergence of the SVIGP algorithm on the two dimensional toy data

land-registry-monthly-price-paid-data/, which covers England and Wales, and filtered for apartments. This resulted in a data set with 75,000 entries,



Figure 5: Variability of apartment price (logarithmically!) throughout England and Wales.

tad a CD with the same commission function as our

### What's Changed (Changing) for Medicine?

Try Googling for: "patient data "...



#### Image from Wikimedia Commons



#### Image from Wikimedia Commons





A brief history of Registration

For more information go to: www.direct.gov.uk/motoring

#### A brief history of registration

#### The early days

Prior to the appearance of the first railways in Britain, there was a brief development and interest in steam powered road going vehicles. In 1834, a Mr Hancock started a steam coach called the "Era", carrying up to 14 passengers from Paddington to Regents Park and the City at 6d a head. And in the following year, a Mr Church built an omnibus capable of carrying 40 passengers for the London and Birmingham Steam Carriage Company.

However, the success of the railway movement drove all such traffic off the roads. A Parliamentary Commission of Enquiry in 1836 reported "strongly in favour of steam carriages on roads", but subsequent Acts of Parliament tended to have a discouraging and restrictive effect. The Locomotive Act 1861 limited the weight of steam engines to 12 tons and imposed a speed limit of 10 mph.

The Locomotive Act 1865 set a speed limit of 4 mph in the country and 2 mph in towns. The 1865 Act also provided for the famous "man with a red flag". Walking 60 yards ahead of each vehicle, a man with a red flag or lantern enforced a walking pace, and warned horse riders and horse drawn traffic of the approach of a self propelled machine.

The Locomotive Amendment Act 1878 made the red flag optional under local regulations, and



Image from Wikimedia Commons



Imaga from Willimodia Commona

### What's Changed (Changing) for Medicine?

- Genotyping.
- Epigenotyping.
- Transcriptome: detailed characterization of phenotype.
  - Stratification of data.

# Open Data

- Automatic data curation: from curated data to curation of publicly available data.
- Open Data: http://www.openstreetmap.org/?lat=53. 38086&lon=-1.48545&zoom=17&layers=M.

· Cocial naturally data music information (Cratify) avancies

# Open Data

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- UK Goverment Stipulation on Data Availability Telegraph Article
- Patient Access: http://www.patient.co.uk/patient-access.asp
- ► The midata project: Tescos, T-mobile ...
- A social network for personal health?? e.g. EMIS myHealth



#### Health

#### Data Heterogenity

Deep Learning

Conclusions

- If missing at random it can be marginalized.
- As data sets become very large (39 million in EMIS) data becomes extremely sparse.
- Imputation becomes impractical.

- Expectation Maximization (EM) is gold standard imputation algorithm.
- Exact EM optimizes the log likelihood.
- Approximate EM optimizes a lower bound on log likelihood.
  - e.g. variational approximations (VIBES, Infer.net).
- Convergence is *guaranteed* to a local maxima in log likelihood.

#### Require: An initial guess for missing data

# **Require:** An initial guess for missing data repeat

# **Require:** An initial guess for missing data **repeat** Update model parameters

(M-step)

Require: An initial guess for missing data repeat Update model parameters Update guess of missing data

(M-step) (E-step) Require: An initial guess for missing data repeat Update model parameters Update guess of missing data until convergence

(M-step) (E-step)

- In very sparse data imputation is impractical.
- EMIS: 39 million patients, thousands of tests.
- For most people, most tests are missing.
- M-step becomes confused by poor imputation.

#### Direct Marginalization is the Answer

Perhaps we need joint distribution of two test outcomes,

 $p(y_1,y_2)$ 

Obtained through marginalizing over all missing data,

$$p(y_1, y_2) = \int p(y_1, y_2, y_3, \dots, y_p) dy_3, \dots dy_p$$

• Where  $y_3, \ldots, y_p$  contains:

- 1. all tests not applied to this patient
- 2. all tests not yet invented!!

#### **Multi-variate Gaussians**

- Given 10 dimensional multivariate Gaussian,  $\mathbf{y} \sim \mathcal{N}(\mathbf{0}, \mathbf{C})$ .
- Generate a single correlated sample  $\mathbf{y} = [y_1, y_2 \dots y_{10}]$ .
- ► How do we find the marginal distribution of *y*<sub>1</sub>, *y*<sub>2</sub>?



(a) A 10 dimensional sample

(b) colormap showing covariance between dimensions.



(a) A 10 dimensional sample

(b) colormap showing covariance between dimensions.



(a) A 10 dimensional sample



(b) colormap showing covariance between dimensions.



(a) A 10 dimensional sample



(b) colormap showing covariance between dimensions.



(a) A 10 dimensional sample



(b) colormap showing covariance between dimensions.



(a) A 10 dimensional sample



(b) colormap showing covariance between dimensions.



(a) A 10 dimensional sample

(b) colormap showing covariance between dimensions.


#### Gaussian Marginalization Property



(a) A 10 dimensional sample

(b) covariance between  $y_1$  and  $y_2$ .

Figure : A sample from a 10 dimensional correlated Gaussian distribution.

#### Gaussian Marginalization Property



(a) A 10 dimensional sample

(b) correlation between  $y_1$  and  $y_2$ .

Figure : A sample from a 10 dimensional correlated Gaussian distribution.

## Avoid Imputation: Marginalize Directly



- Our approach: Avoid Imputation, Marginalize Directly.
- Explored in context of Collaborative Filtering.
- Similar challenges:
  - many users (patients),
  - many items (tests),
  - sparse data
- Implicitly marginalizes over all future tests too.

Work with Raquel Urtasun (Lawrence and Urtasun, 2009) and recent submission with Nicoló Fusi.

#### Methods that Interrelate Covariates

- Need Class of models that interrelates data.
- Common assumption: high dimensional data lies on low dimensional manifold.
- Want to retain the marginalization property of Gaussians.

#### Linear Dimensionality Reduction

#### Linear Latent Variable Model

- Represent data, Y, with a lower dimensional set of latent variables X.
- Assume a linear relationship of the form

$$\mathbf{y}_{i,:} = \mathbf{W}\mathbf{x}_{i,:} + \boldsymbol{\epsilon}_{i,:},$$

where

$$\boldsymbol{\epsilon}_{i,:} \sim \mathcal{N}\left(\mathbf{0}, \sigma^2 \mathbf{I}\right).$$

#### Linear Latent Variable Model II

#### Probabilistic PCA Max. Likelihood Soln (Tipping and Bishop, 1999)



$$p(\mathbf{Y}|\mathbf{W}) = \prod_{i=1}^{n} \mathcal{N}\left(\mathbf{y}_{i,:}|\mathbf{0}, \mathbf{W}\mathbf{W}^{\top} + \sigma^{2}\mathbf{I}\right)$$

#### Linear Latent Variable Model II

Probabilistic PCA Max. Likelihood Soln (Tipping and Bishop, 1999)

$$p(\mathbf{Y}|\mathbf{W}) = \prod_{i=1}^{n} \mathcal{N}(\mathbf{y}_{i,:}|\mathbf{0}, \mathbf{C}), \quad \mathbf{C} = \mathbf{W}\mathbf{W}^{\top} + \sigma^{2}\mathbf{I}$$
$$\log p(\mathbf{Y}|\mathbf{W}) = -\frac{n}{2}\log|\mathbf{C}| - \frac{1}{2}\mathrm{tr}\left(\mathbf{C}^{-1}\mathbf{Y}^{\top}\mathbf{Y}\right) + \mathrm{const.}$$

If  $\mathbf{U}_q$  are first q principal eigenvectors of  $n^{-1}\mathbf{Y}^{\mathsf{T}}\mathbf{Y}$  and the corresponding eigenvalues are  $\mathbf{\Lambda}_q$ ,

$$\mathbf{W} = \mathbf{U}_q \mathbf{L} \mathbf{R}^{\mathsf{T}}, \quad \mathbf{L} = \left( \mathbf{\Lambda}_q - \sigma^2 \mathbf{I} \right)^{\frac{1}{2}}$$

where **R** is an arbitrary rotation matrix.

#### Dealing with Non Gaussian Data

- Marginalization property of Gaussians very attractive.
- How to incorporate non-Gaussian data?
  - Data which isn't missing at random.
  - Binary data.
  - Ordinal categorical data.
  - Poisson counts.
  - Outliers.

## Project Back into Gaussian

- Combine non-Gaussian likelihood with Gaussian prior.
- Either:
  - Project back to Gaussian posterior that is nearest in KL sense.
  - Expectation propagation.
- ► Or:
  - Fit a locally valid Gaussian approximation.
  - Laplace Approximation.

Ongoing work with Ricardo Andrade Pacheco (EP) and Alan Saul (Laplace) also James Hensman.





#### Gaussian Noise



Figure : Inclusion of a data point with Gaussian noise.

#### Gaussian Noise



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#### Gaussian Noise



Figure : Inclusion of a data point with Gaussian noise.

#### **Classification Noise Model**

#### Probit Noise Model



Figure : The probit model (classification). The plot shows  $p(y_i|f_i)$  for different values of  $y_i$ . For  $y_i = 1$  we have  $p(y_i|f_i) = \phi(f_i) = \int_{-\infty}^{f_i} \mathcal{N}(z|0, 1) dz$ .



Figure : An EP style update with a classification noise model.



Figure : An EP style update with a classification noise model.



Figure : An EP style update with a classification noise model.



Figure : An EP style update with a classification noise model.

#### Ordinal Noise Model

**Ordered Categories** 



Figure : The ordered categorical noise model (ordinal regression). The plot shows  $p(y_i|f_i)$  for different values of  $y_i$ . Here we have assumed three categories.



Figure : An EP style update with an ordered category noise model.



Figure : An EP style update with an ordered category noise model.



Figure : An EP style update with an ordered category noise model.



Figure : An EP style update with an ordered category noise model.

- Spatial Data (workshop in November with Peter Diggle, work with Ricardo Andrade Pacheco and John Quinn's group).
- Survival Data (work with Alan Saul and Aki Vehtari's group and HeRC).
- Image Data (work with Teo de Campos, Violet Snell and imminent arrival of Zhenwen Dai)
- Text Data (planned collaboration with Trevor Cohn)

## Example: Prediction of Malaria Incidence in Uganda

- Work with John Quinn and Martin Mubaganzi (Makerere University, Uganda)
- See http://cit.mak.ac.ug/cs/aigroup/.

# Malaria Prediction in Uganda



Data SRTM/NASA from http://dds.cr.usgs.gov/srtm/version2\_1



#### Malaria Prediction in Uganda



## Malaria Prediction in Uganda



# Visit to Uganda





Health

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WebmasterWorld Forums





direction for further research.

#### 11.1. HAVE WE THROWN THE BABY OUT WITH THE BATH WATER?

According to the hype of 1987, neural networks were meant to be intelligent models which discovered features and patterns in data. Gaussian processes in contrast are simply smoothing devices. How can Gaussian processes possibly replace neural networks? What is going on?

I think what the work of Williams and Rasmussen (1996) shows is that many real-world data modelling problems are perfectly well solved by sensible smoothing methods. The most interesting problems, the task of feature discovery for example, are not ones which Gaussian processes will solve. But maybe multilayer perceptrons can't solve them either. On the other hand, it may be that the limit of an infinite number of hidden units, to which Gaussian processes correspond, was a bad limit to take; maybe we should backtrack, or modify the prior on neural network parameters, so as to create new models more interesting than Gaussian processes. Evidence that this infinite limit has lost something compared with finite neural networks comes from the observation that in a finite neural network with more than one output, there are non-trivial correlations between the outputs (since they share inputs from common hidden units); but in the limit of an infinite number of hidden units, these correlations vanish. Radford Neal has suggested the use of non-Gaussian priors in networks with multiple hidden layers. Or perhaps a completely fresh start is needed, approaching the problem of machine learning from a paradigm different from the supervised feedforward mapping.

MacKay: NIPS Tutorial 1997 "Have we thrown out the baby with the bathwater?" (Published as MacKay, 1998) Also noted by (Wilson et al., 2012)
## Deep Models



## Deep Models



# Deep Models



### Deep Gaussian Processes



Damianou and Lawrence (2013)

- Deep architectures allow abstraction of features (Bengio, 2009; Hinton and Osindero, 2006; Salakhutdinov and Murray, 2008).
- We use variational approach to stack GP models.































### What Can We Do that Google Can't?

- Google's resources give them access to volumes of data (or Facebook, or Microsoft, or Amazon).
- Is there anything for Universities to contribute?
- Assimilation of multiple views of the patient: each perhaps from a different patient.
- This may be done by small companies (with support of Universities).
- A Facebook app for your personalised health.
- These methodologies are part of that picture.

# Deep Health



## Deep Health: Power Ranger Model of Research



## GPy: A Gaussian Process Framework in Python

- BSD Licensed software base.
- Wide availability of libraries, "modern" scripting language.
- Allows us to set projects to undergraduates in Comp Sci that use GPs.
- Available through GitHub https://github.com/SheffieldML/GPy
- Reproducible Research with IPython Notebook.

#### Probable Features for Next Release

- Non-Gaussian likelihoods.
- Multivariate outputs.
- Dimensionality reduction.
- Approximations for large data sets.
- Probabilistic-style programming (specify the model, not the algorithm).
- A range of covariance functions, ability to implement your own symbolically.
- Data missing at random.

#### Planned Features for Future Releases

- Deep models.
- Massive data: millions of points.
- Data missing not at random.
- Deep models for massive data.

### GPSS: Gaussian Process Summer School



- http://ml.dcs.shef.ac.uk/gpss/
- Next one is a Winter School in Sheffield 13th-15th January 2014.
- To be followed by a workshop on Temporospatial Modelling (16th January).
- Ran a 'Gaussian Process Roadshow' at Makerere in Uganda this August.
- Will run a Roadshow in Colombia in February.
- Tentative plans for a Roadshow in Kenya in 2015.



- Gaussian models good for missing data.
- Disparate data types handled with EP and Laplace.
- Deep models allow complex abstract representation of data sets at higher levels.
- Current limitation is on data set size.
- Addressing this through work by James Hensman on Stochastic Variational Inference for GPs (recent UAI paper).
- Intention is to deploy these models for assimilating a wide range of data types in personalized health (text, survival times, images, genotype, phenotype).
- Requires population scale models with millions of features.

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