

Discrete Foundations — Question Sheet 1

Please complete this by Friday 8 October, when the solutions will be given. The starred question (question 7) is primarily for COM6853 students.

1. Let $U = \{1, 2, \dots, 8, 9\}$, and

$$\begin{aligned} A &= \{1, 2, 3, 4, 5\}, & C &= \{5, 6, 7, 8, 9\}, & E &= \{2, 4, 6, 8\}, \\ B &= \{4, 5, 6, 7\}, & D &= \{1, 3, 5, 7, 9\}, & F &= \{1, 5, 9\}. \end{aligned}$$

Find (a) $A \cup B$, (b) $A \cap B$, (c) $B \cup D$, (d) $A \cup C$, (e) $A \cap C$, (f) $D \cup F$, (g) $A \setminus B$, (h) $B \setminus A$, (i) $D \setminus E$, (j) $F \setminus D$, (k) $A \cap (B \cup E)$, (l) $(A \cap D) \setminus B$, (m) $(A \setminus E)^c$, (n) $(B \cap F) \cup (C \cap E)$.

2. Draw the Venn diagrams of the sets $A \cap B^c$ and $(B \setminus A)^c$.

3. Use the laws of sets to prove the identity $(U \cap A) \cup (B \cap A) = A$.

4. In a survey of 60 people, it was found that 25 people read *Newsweek*, 26 people read *Time*, and 26 people read *Fortune* magazine. Also, 9 people read both *Newsweek* and *Fortune*, 11 people read both *Newsweek* and *Time*, 8 people read both *Time* and *Fortune*, and 8 people do not read any of the magazines. Use a Venn diagram to calculate the number of people who read all three magazines, and the number of people who read only one magazine.

5. Which of the following statements are true and which are false ?

$$\begin{aligned} 1. \emptyset \subseteq \{\text{cat}, \text{dog}\} & \quad 2. \{\text{cat}\} \not\subseteq \{\text{cat}, \text{dog}\} & \quad 3. \{\text{cat}, \text{dog}\} \subset \{\text{cat}, \text{dog}\} \\ 4. \{\text{cat}, \text{dog}\} \not\subseteq \emptyset & \quad 5. \{\text{cat}, \text{dog}\} \subset \{\text{cat}\} \end{aligned}$$

6. Let $S = \{a, b, c\}$. Which of the following statements are true and which false?

$$1. a \subseteq S \quad 2. d \subseteq S \quad 3. \emptyset \subseteq S \quad 4. \{d\} \not\subseteq S$$

7*. We define the *symmetric difference* $A \Delta B$ of two sets A, B by

$$A \Delta B = (A \setminus B) \cup (B \setminus A).$$

1. Draw a Venn diagram showing typical sets A, B with $A \Delta B$ shaded.

2. Draw Venn diagrams with typical sets A, B, C , illustrating the identity

$$(A \Delta B) \Delta C = A \Delta (B \Delta C). \tag{1}$$

3. Prove (1) from the algebraic laws for sets. Hint: show that each side is equal to

$$(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C) \cup (A \cap B \cap C).$$