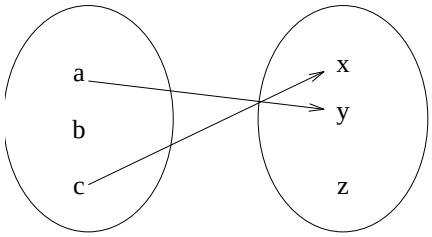
Discrete Foundations — Question Sheet 2

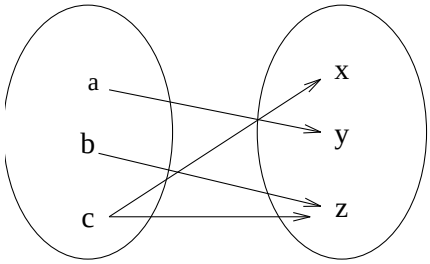
Please complete this by Friday 15 October, when the solutions will be given. The starred question is for PMA6853 students.

1. Determine if each of the diagrams below defines a function from $A = \{a, b, c\}$ to $B = \{x, y, z\}$.

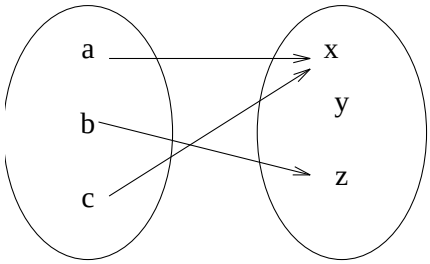
(a)



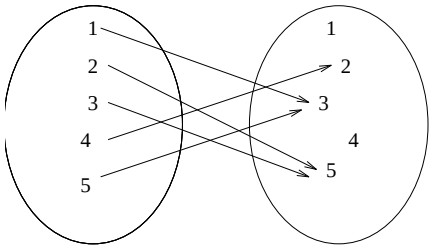
(b)



(c)



2. Let $A = \{1, 2, 3, 4, 5\}$ and let $f : A \to A$ be the function defined in the diagram below. Find $f(A)$, i.e. the range of f.



3. Let A be the set of students in a school. Determine which of the following assignments defines a function on A.
(a) To each student, assign his/her age.
(b) To each student, assign his/her teacher.
(c) To each student, assign his/her sex.
(d) To each student, assign his/her spouse.

4. A function $f : A \to B$ is *one–to–one* (written 1–1) if different elements in the domain of A have distinct images, that is, f is *one–to-one* if $f(a) = f(b)$ implies that $a = b$. Determine if each of the following functions is 1–1.
(a) To each person on the earth, assign the number which corresponds to his age.
(b) To each country in the world, assign the latitude and longitude of its capital.
(c) To each book written, assign the (first-named) author.
(d) To each country in the world that has a prime minister, assign its prime minister.

5. Let $S = \{1, 2, 3, 4, 5\}$ and consider the following functions from S to S : $f(n) = n$, $g(n) = 6 - n$, $h(n) = \max\{3, n\}$ and $l(n) = \max\{1, n - 1\}$. Which of these functions are 1–1, onto ?

6. Determine which of the following functions are injective and which are surjective :
(a) $f : \mathbb{Z} \to \mathbb{N}$ where $f(x) = x^2 + 1$
(b) $g : \mathbb{N} \to \mathbb{N}$ where $g(n) = 2^n$
(c) $h : \mathbb{R} \to \mathbb{R}$ where $h(x) = 5x - 1$
(d) $f : \mathbb{R} \to \mathbb{R}$ where

$$f(x) = \begin{cases} 2x - 3 & \text{if } x \geq 1 \\ x + 1 & \text{if } x < 1 \end{cases}$$

(e) $k : \mathbb{R} \to \mathbb{R}$ where $k(x) = x + |x|$
(f) $l : \mathbb{R} \to \mathbb{R}$ where $l(x) = 2x - |x|$
Note : $|x|$ denotes the absolute value of x. Thus $|x| = x$ if $x \geq 0$ and $|x| = -x$ if $x < 0$.

7. Write down the Cartesian products $A \times B$ and $B \times A$ of the sets $A = \{0, 1, 2\}$ and $B = \{0\}$.

8.* Let $f : X \to Y$ be a function. For $A \subseteq X$ we write $f(A) = \{f(a) : a \in A\}$ and for $G \subseteq Y$ we write $f^{-1}(G) = \{x \in X : f(x) \in G\}$. [I have used the conventional notation here, but notice that $f^{-1}(G)$ is defined even when f is not invertible.] Which of the following general statements are true:
(a) $f(A \cup B) = f(A) \cup f(B)$ for all $A, B \subseteq X$;
(b) $f(A \cap B) = f(A) \cap f(B)$ for all $A, B \subseteq X$;
(c) $f^{-1}(G \cup H) = f^{-1}(G) \cup f^{-1}(H)$ for all $G, H \subseteq Y$;
(d) $f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H)$ for all $G, H \subseteq Y$?
Justify your answers. When '=' is not true, consider whether '$\subseteq$' or '$\supseteq$' might be true.