COM3502/4502/6502
SPEECH PROCESSING

Lecture 12
Frequency Analysis

Time vs. Frequency Domain

- A signal may represent a sequence of *any* kind of measurement
- It is common for signals to represent a sequence of measurements in *time*
- A useful technique for signal analysis is to decompose it into a set of basic pieces that are easier to understand (*e.g. linear trends*)
- Decomposing into *periodic* functions can be thought of as transforming a signal into the 'frequency domain' (*i.e. repetitions in time can be expressed as a frequency*)

$f = 1/\tau$ cycles-per-second/Hertz

Fourier Analysis

- Introduced by **Joseph Fourier** (*1768–1830*)
- **Theorem**: any *periodic* signal of frequency f_0 can be constructed exactly by adding together 'sinusoids' (*sine waves*) with frequencies …
  - f_0, $2f_0$, $3f_0$, $4f_0$, $5f_0$, etc.
- Each sinusoid in this 'Fourier series' is characterised by its …
  - frequency
  - amplitude
  - phase
- f_0 is termed the 'fundamental frequency'
- $2f_0$, $3f_0$, $4f_0$, etc. are termed the 'harmonics'

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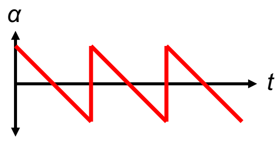
Fourier Analysis

$$a(t) = \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\sin(k\omega t)}{k}$$

$$= \frac{2}{\pi}(\sin(\omega t) + \frac{\sin(2\omega t)}{2} + \frac{\sin(3\omega t)}{3} + \ldots)$$

All harmonics

'sawtooth' waveform



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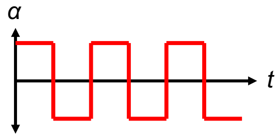
Fourier Analysis

$$a(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin((2k-1)\omega t)}{(2k-1)}$$

$$= \frac{4}{\pi}(\sin(\omega t) + \frac{\sin(3\omega t)}{3} + \frac{\sin(5\omega t)}{5} + \ldots)$$

Odd harmonics

'squarewave'



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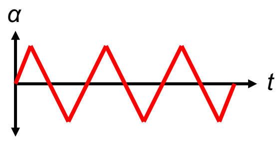
Fourier Analysis

$$a(t) = \frac{8}{\pi^2}\sum_{k=0}^{\infty}(-1)^k\frac{\sin((2k+1)\omega t)}{(2k+1)^2}$$

$$= \frac{8}{\pi^2}(\sin(\omega t) - \frac{\sin(3\omega t)}{9} + \frac{\sin(5\omega t)}{25} - \ldots)$$

Odd harmonics
(alternating sign)

'trangular wave'



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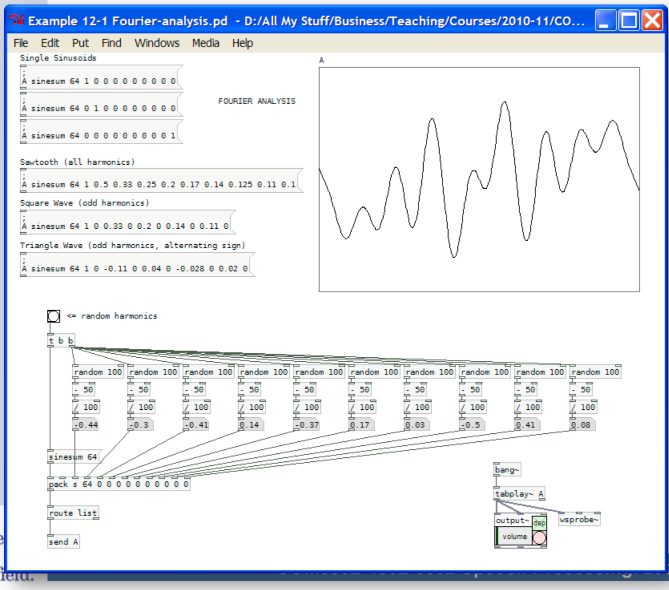
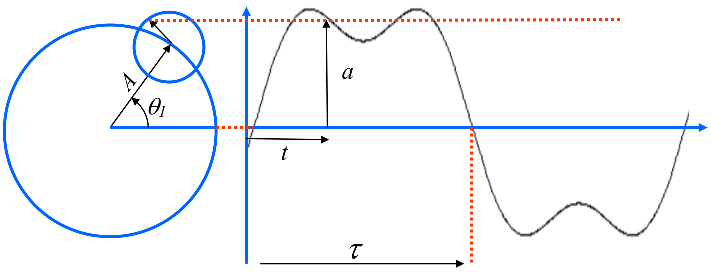
Fourier Analysis



Fourier Analysis

Projection of multiple vectors rotating at constant velocities

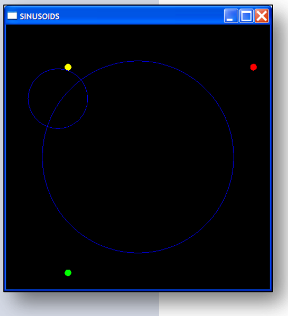


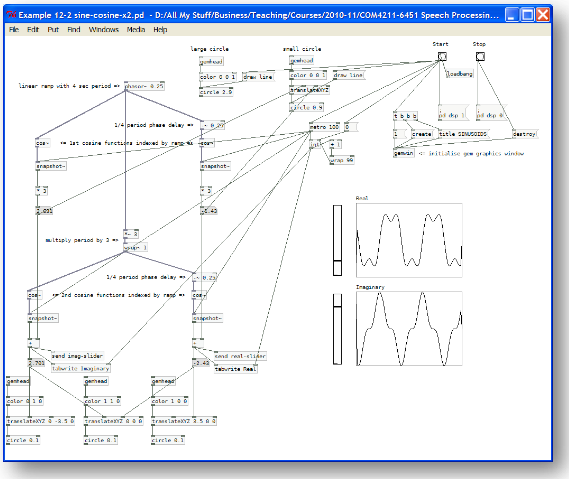
A	magnitude of 1st rotating vector
θ	phase angle of 1st vector (*radians*)
$f_0 = 1/\tau$	fundamental frequency (*cycles per second, Hertz*)
$\omega = 2\pi f_0$	fundamental angular frequency (*radians/second*)

$\theta = \omega t$

$a = A.\sin(\omega t) + A/3.\sin(3\omega t)$

Fourier Analysis

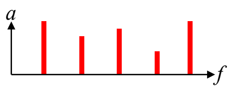




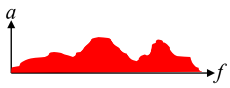
The Spectrum

- Any periodic function can be characterised by the amplitude and phase of its sinusoidal components – it's 'spectrum'

- Periodic signals only have spectral components at *integer multiples* of the fundamental (*c.f. Fourier's theorem*)

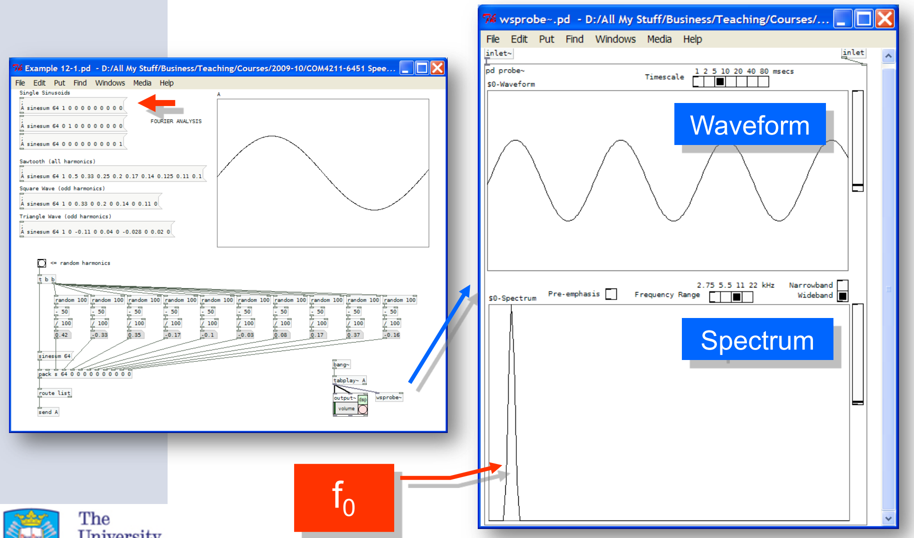


- Aperiodic and stochastic signals have spectra that are continuous functions of frequency (*i.e. all possible frequencies*)



- White noise has a *flat* spectrum (*i.e. equal energy at all frequencies*)

The Spectrum



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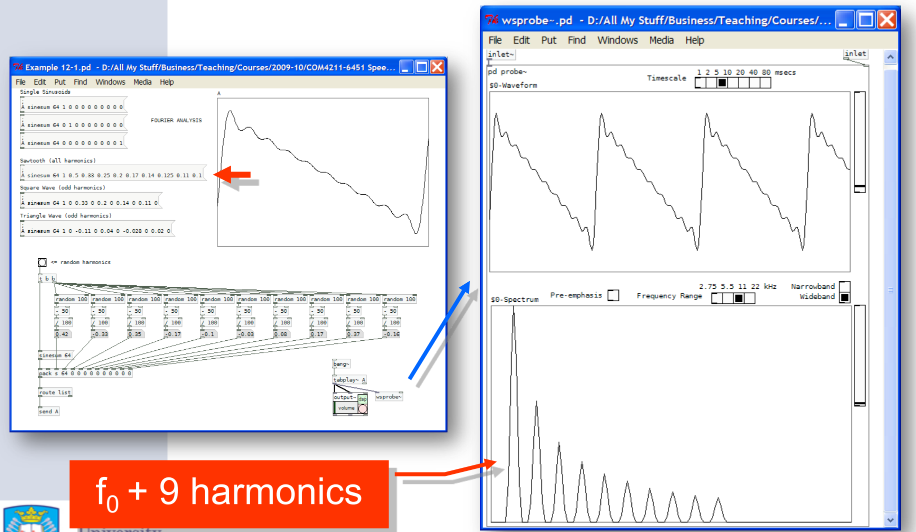
The Spectrum



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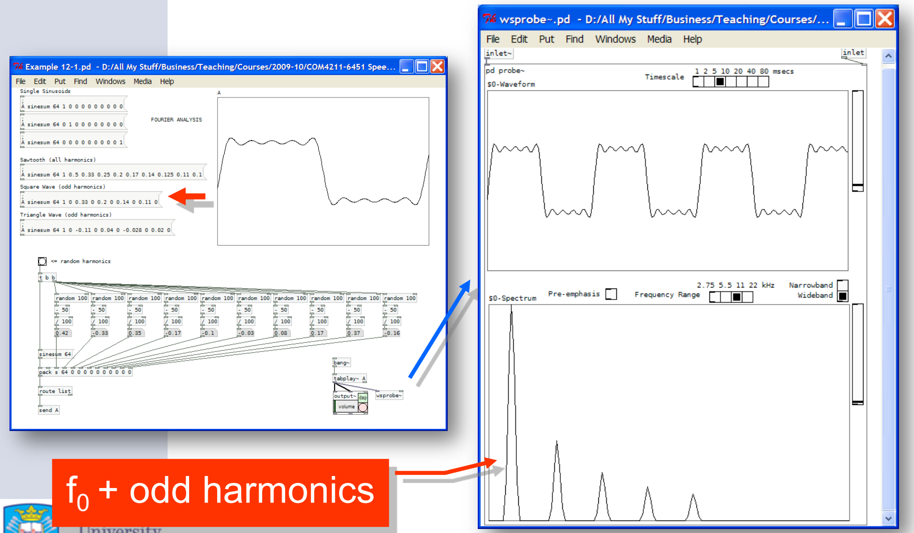
The Spectrum



f_0 + odd harmonics

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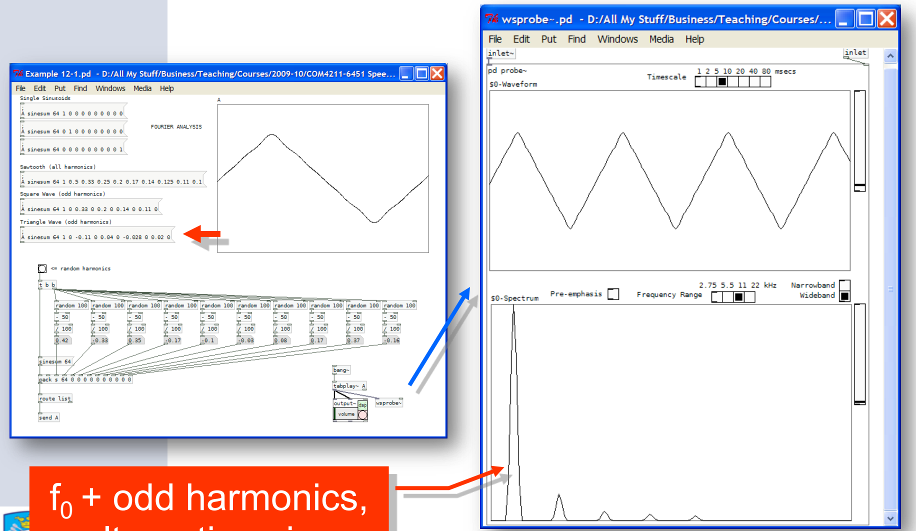
The Spectrum



f_0 + odd harmonics, alternating sign

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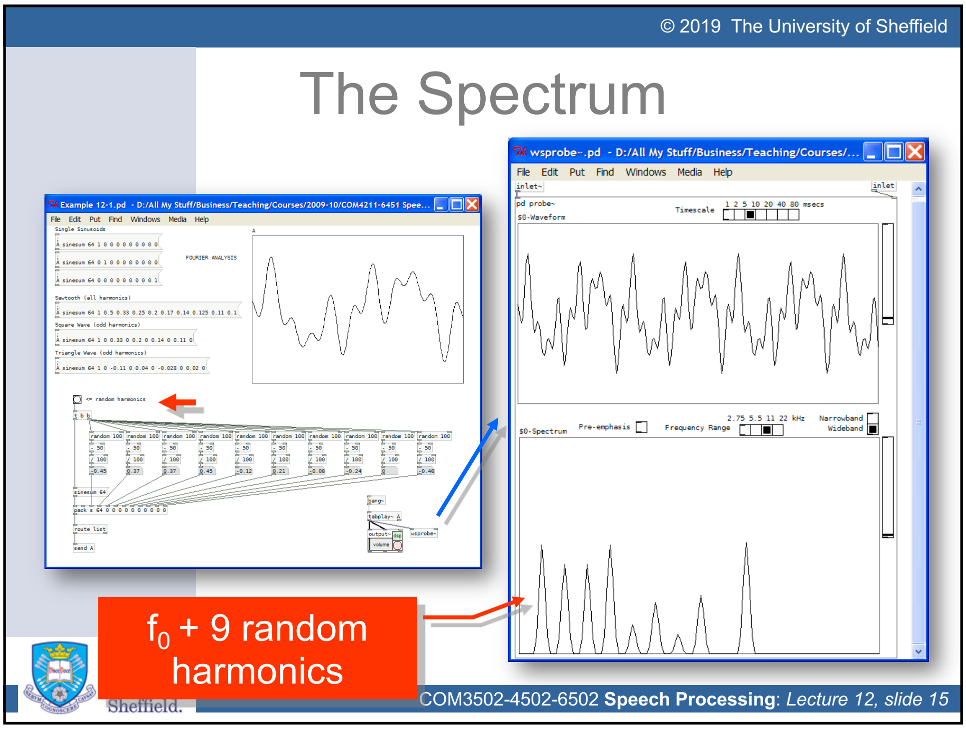
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The Spectrum
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f0 + 9 random harmonics
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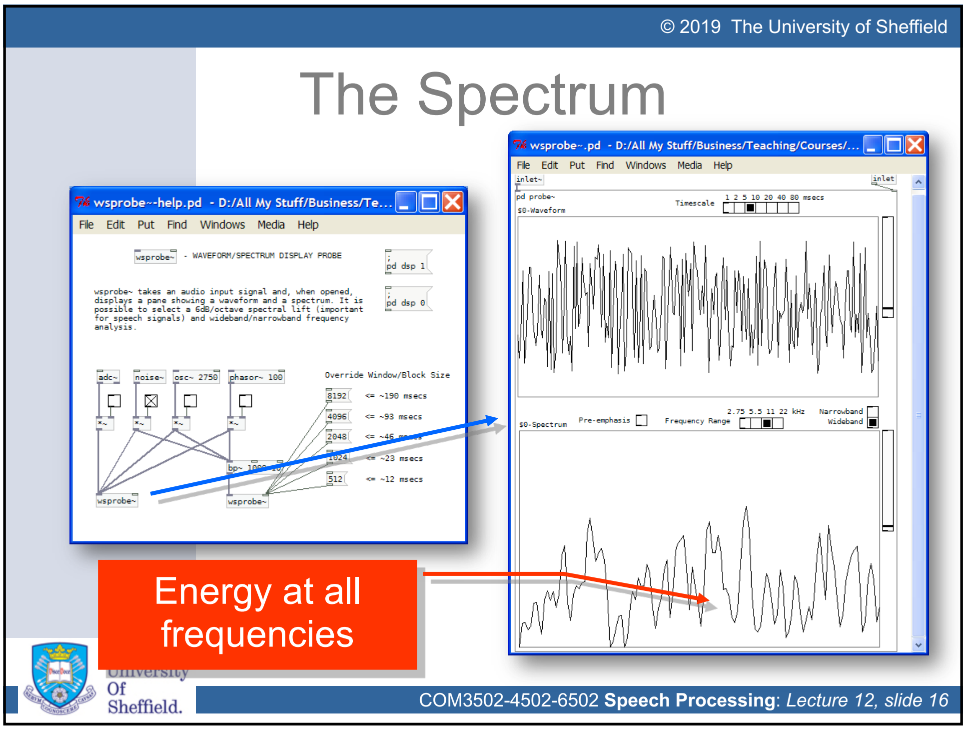
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The Spectrum
wsprobe~-help.pd - D:/All My Stuff/Business/Te...
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Energy at all frequencies
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The Power Spectrum

- Previous examples have illustrated the '**power spectrum**', i.e. the energy present at each frequency
- The power of a signal at each frequency is the sum of the squares of its real and imaginary components (*e.g. a sine wave has varying amplitude but constant power*)
- Information (*the phase relationships*) is lost during the calculation of the power spectrum, so this means that the original signal cannot be recovered accurately
- However, a signal's spectrum is fully specified by the magnitude and phase at each frequency … this is the '**complex spectrum**'

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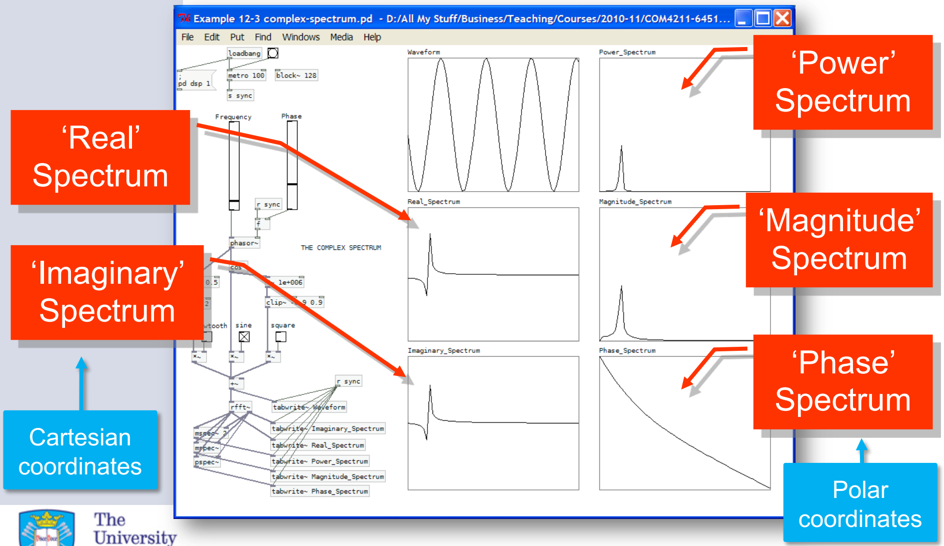
The Complex Spectrum



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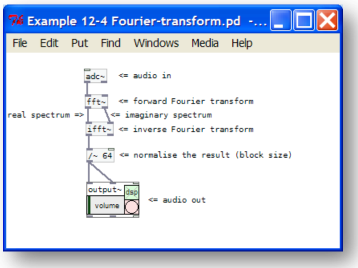
The Fourier Transform

- The complex (magnitude and phase) spectrum is obtained from a time signal by means of the 'Fourier transform'
- No information is lost during the transformation
- This means that the time signal can be recovered from the complex spectrum by means of the 'inverse Fourier transform'



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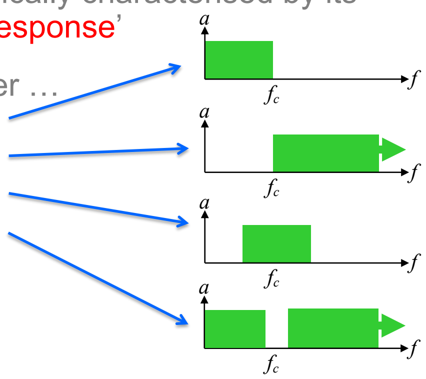
Filters

- A 'filter' modifies the properties of a signal
- A filter's effect can be analysed in the time domain or in the frequency domain
- A filter is typically characterised by its 'frequency response'
- Types of filter …
  - low-pass
  - high-pass
  - band-pass
  - band-stop



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Filters

- The '**frequency response**' of a *low-pass* or a *high-pass* filter is characterised by …
  - the '**cut-off frequency**' f_c (*Hz*)
  - the '**roll-off rate**' (*dB/octave*)
- A '**first-order filter**' attenuates at a roll-off rate of 6 dB/octave
- The frequency response of a *band-pass* filter is characterised by
  - the '**centre frequency**' f_c (*Hz*)
  - the '**bandwidth**' (*Hz*) or '**Q**' (=*cf/bw*)
- When a signal passes through a linear filter, its spectrum is *multiplied* by the frequency response of the filter

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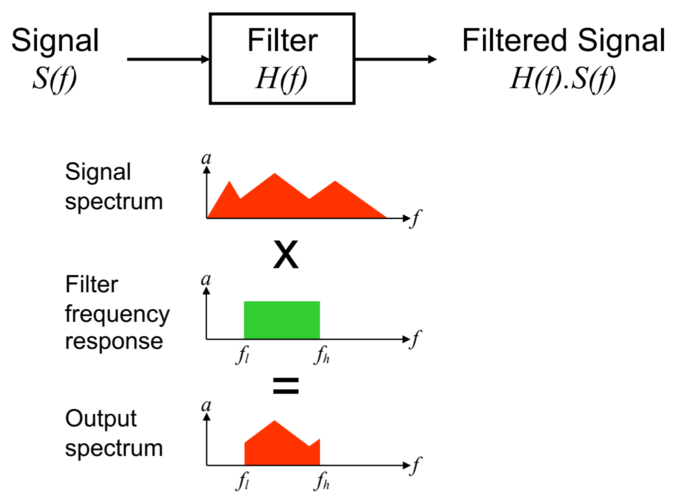
Filters



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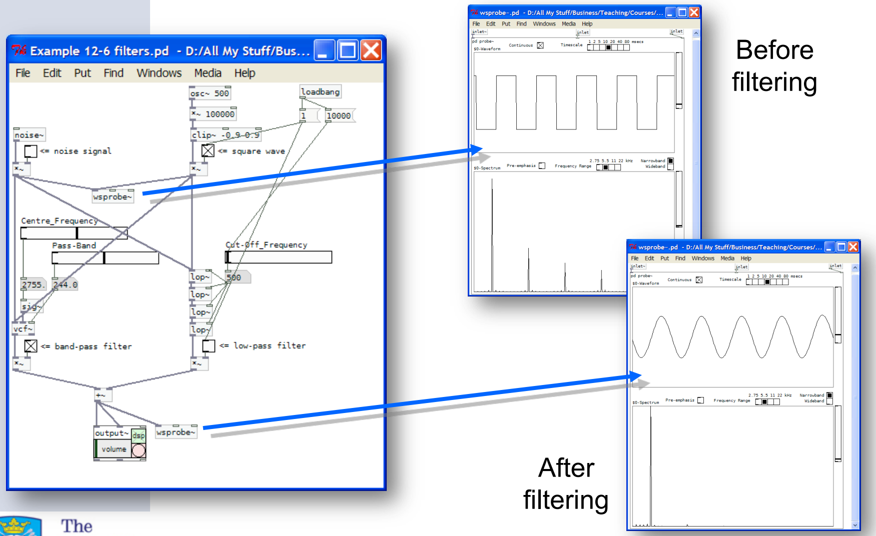
Filters



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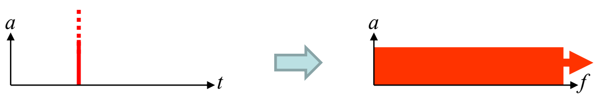
Impulse Response

- An 'impulse' signal is an instantaneous pulse of energy
- An idealised impulse signal has a *flat* frequency spectrum (*i.e. equal energy at all frequencies*)



- Hence the 'impulse response' of a filter has a spectrum that is *equal* to its frequency response
- Note that white noise *also* has a flat spectrum

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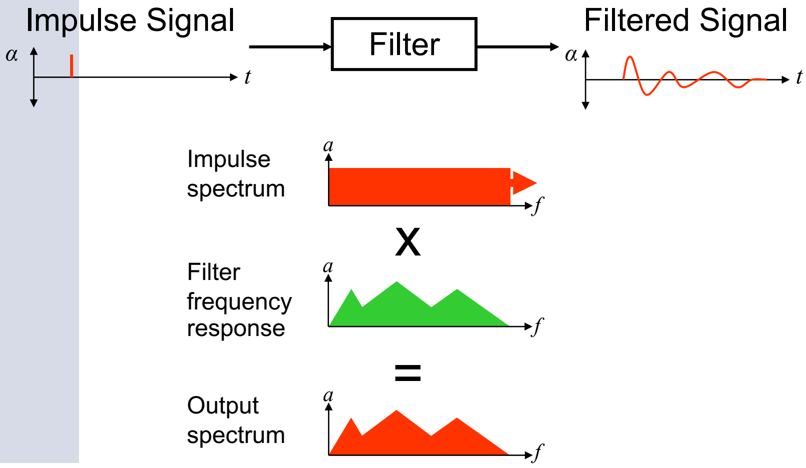
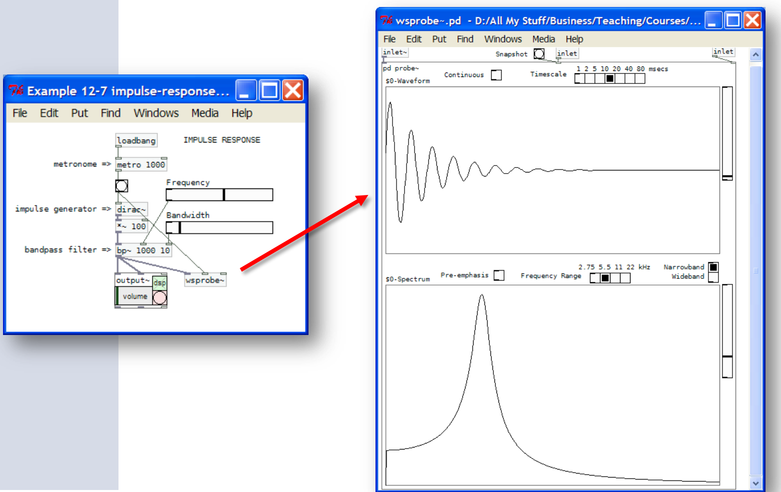
Impulse Response



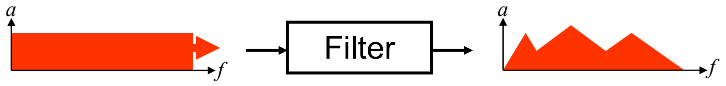
Impulse Response



Source-Filter Modelling

- If we design a filter with a frequency response equal to the spectrum of a vowel sound and input a sequence of impulses, we will generate the original vowel sound …

- Also, if we design a filter with the frequency response of an unvoiced fricative and input a white noise signal, we will generate the original fricative sound …



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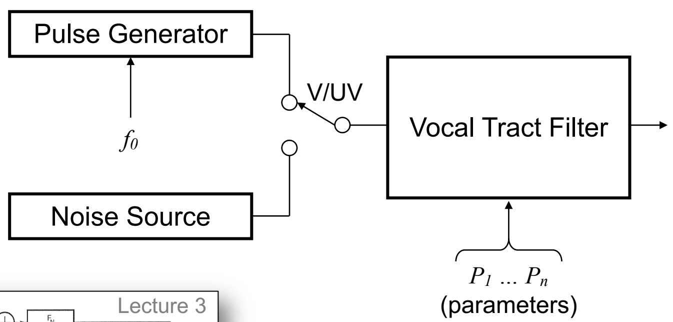
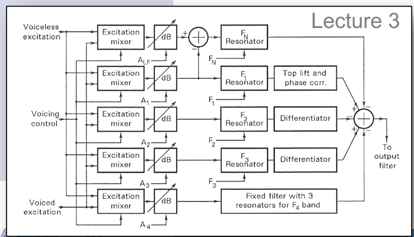
Source-Filter Model of Speech





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This lecture has covered …

- Fourier analysis
- Fundamental frequency
- Harmonics
- The power spectrum
- The complex spectrum
- The Fourier transform
- Filters
- Impulse response
- The source-filter model of speech

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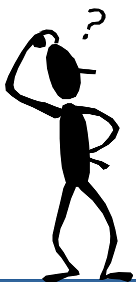
Any Questions ?



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MOLE Quiz #1

- 16:00 Wednesday 13th November in North Campus PC room
- 20 questions (*multiple-choice, true/false, etc.*)
- Covers material from Lecture 1 to Lecture 10
- Worth 20% of the overall course mark
- 1hr in duration (*longer if you have an LSP in place*)
- You may start the test anytime up to the final deadline
- Only one attempt is permitted
- You may save your work part way through, but the clock will keep running
- You may continue after the time expires, but the system will record that you took more than the allocated time

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MOLE Quiz #1

- Any phonetic symbols are displayed using SAMPA
- The quiz is marked automatically, so avoid making any spelling mistakes (*e.g. it's "vocal cords" not "vocal chords"*)
- This is an individual 'open-book' assessment
(*i.e. you may refer to your course notes, but you must not collude with anyone during the test*)
- Marks will appear immediately after submission
- Feedback (*i.e. the correct answers*) will appear one week after the deadline
- You may pack-up and leave when you are finished, but please do so quietly

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Next time …

Digital Signals