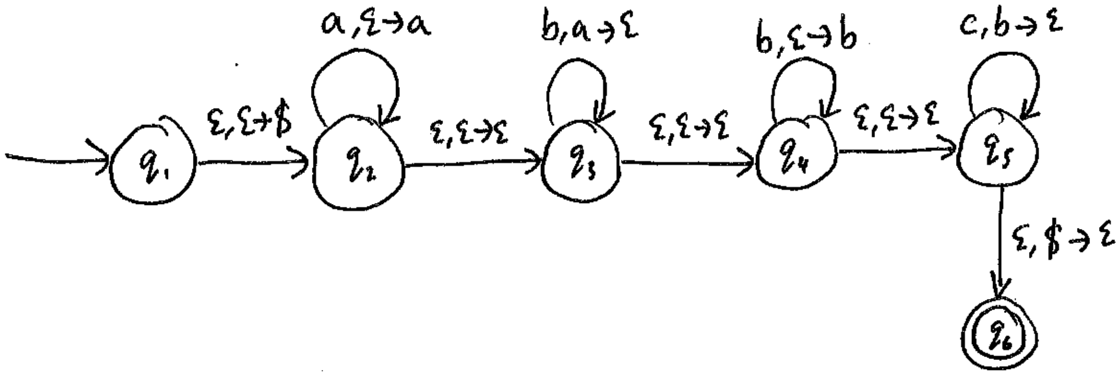
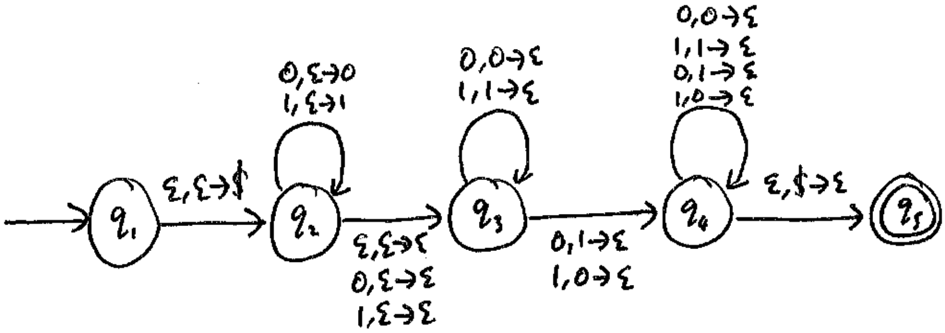
COM 2003
Problem Sheet 2 Solutions

1) a)



b)



2) a) $S \to aSa \mid bSb \mid cSc \mid a \mid b \mid c \mid \varepsilon$

b)
$$S \to aS_1 \mid aS_3$$
$$S_1 \to S_2 \mid S_1c$$
$$S_2 \to aS_2 \mid aS_2b \mid \varepsilon$$
$$S_3 \to aS_3 \mid aS_3c \mid S_4$$
$$S_4 \to bS_4 \mid \varepsilon$$

3) The language is odd-length palindromes.

$$S \to aSa \mid bSb \mid a \mid b$$

→ cont.

4) The language does not ~~looks~~ look context-free; checking that $j>i$ destroys our b-count we need to check that $k>j$.

Thus, we use the pumping lemma, and try to pump the string $a^p b^{p+1} c^{p+2}$:

By condition (3), $|vxy| \leq p$, so we have five possible forms of vxy:

1. a's ⇒ By pumping up, we see that there are more a's than b's.
2. a's & b's ⇒ By pumping up, the order of a's and b's changes.
3. b's ⇒ By pumping up, we see that there are more b's than c's.
4. b's & c's ⇒ By pumping up, the order of b's and c's changes.
5. c's ⇒ By pumping down, there are equal, or less, c's than b's.

Therefore, condition (1) is not satisfied, so the language is not context-free.

5) Suppose, for a contradiction, that a correspondence exists between $\mathbb{N}$ and $\mathcal{P}(\mathbb{N})$. We can describe each member of $\mathcal{P}(\mathbb{N})$ as a binary string ~~e.g.~~ of infinite length, e.g:

$\mathbb{N}$	1	2	3	4	5	6	7	8	9	10 ...
$A \in \mathcal{P}(\mathbb{N})$	0	0	1	1	0	1	0	0	1	0 ...

Now, we can construct a member of $\mathcal{P}(\mathbb{N})$ unpaired with a member of $\mathbb{N}$ by diagonalisation, to reach a contradiction. Alternatively, we can note the identity of $\mathcal{P}(\mathbb{N})$ with the binary strings, which are uncountable.