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Comments on Multimodal Systems

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1 Introduction

The main paper of this deliverable presents a number of interesting ideas and results concerning mixed Lambek systems. The move from simple to mixed Lambek systems constitutes, in my opinion, a crucially important development in the type-logical thread of categorial grammar research. However, the ideas involved in using mixed systems are still quite new, and a number of issues that arise remain to be resolved. The comments made in the next few pages relate specifically with that part of the deliverable that concerns *multimodal logics* (the second type of mixed system presented in the main paper). In particular, I will take issue with certain claims made there concerning the specifics of how the coexisting sublogics of a multimodal system should be interrelated.

I have independently developed proposals for the construction of multimodal systems (Hepple [3]). These proposals begin from a somewhat different starting point to those presented in the main paper, specifically ideas involving ***structural modalities*** and *embedding translations*. Despite this different point of origin, the resulting mixed logic (in Gentzen format) differs only in minor details from that presented in the main paper. The crucial difference that remains between the two approaches concerns the claims that are made of how the different substructural levels should be linked. Specifically, the two approaches make systematically contradictory claims concerning what are the 'natural' directions of linkage between levels, so that, for example, where one system (i.e. that of the main paper) has the theorem $X{\bullet}Y \Rightarrow X{\otimes}Y$, the other system has $X{\otimes}Y \Rightarrow X{\bullet}Y$. It is convenient for the following comments to have 'labels' for the two contrasting views (although naming them in this way tends to obscure the fact that they agree in most regards). I will call the view that I have taken the "hybrid" view, and that presented in the main paper the "M&O" view (for "Moortgat & Oehrle"[1]).

In what follows, I will introduce the alternative 'hybrid' view of multimodal linkage, beginning with the ideas of structural modalities and embedding translations that give rise to it. (Note that where I speak of 'linkage' in these comments, I refer primarily to what the main paper calls inclusions, and not interactions, concerning which no disagreement arises.) In addition, I will mention some differences that follow from the hybrid view of linkage for using a multimodal system as a grammatical formalism, and also discuss the 'general linguistic approach' that the hybrid view tends to foster. Finally, I will again address the two contrasting views, and consider if there is a straightforward answer to whether one view is right and the other wrong.

Note that these comments are intended to be read in conjunction with the main paper, not independently. Consequently I omit much in the way of introductory comment, statement of rules, references, etc., that would otherwise be needed.[2] The examples that I will use in these comments

[1] The multimodal ideas of the main paper are elsewhere presented in Moortgat & Oehrle [5, 6].

[2] It may be useful to point out some of the key things that I assume from the main paper: (i) the Gentzen presentation of a unimodal logic for a generic product and its residuals, (ii) structural rules, and their role in determining the character of resource sensitivity, (iii) the systems **L**, **NL**, **LP** and **NLP**, and the structural rule choices that give rise to them, (iv) the basic ideas of frame semantics, multimodal frames, and inclusion and interaction principles, (v) the Gentzen format presentation of a general multimodal system, including mode specific structural rules and 'linkage' rules.

will all involve the systems **L**, **NL** and **LP**. For convenience, I will use the following specific notations for the products and implications of these systems: **L**:$\{\bullet, \backslash, /\}$, **NL**:$\{\odot, \not\backslash, \not/\}$, **LP**:$\{\otimes, \multimap, \circ\!-\}$.

2 Categorial grammar and structural modalities

Earlier work with categorial systems based at just a single level of the substructural hierarchy suggests that such systems are of restricted value for linguistic analysis, because of the complexities of resource usage to be found within any individual language, and, more generally, the differences of resource usage found between languages. Such limitations suggests the need of systems that allow exploitation of the resource usage characteristics of more than one substructural level.

One approach to realising this goal has employed *structural modalities*, which are unary operators that allow controlled involvement of structural rules that are not otherwise available in a system.[3] In constructing a linguistic system under this approach, some specific resource logic must first be chosen as 'basic' for stating the grammar, thereby setting the *default* characteristics of resource sensitivity. Then, structural modalities are used to allow controlled access to the resource sensitivity of higher substructural levels. Various theoretical, computational and practical problems arise for the use of such operators. One consideration is that the need to have a single 'base' logic, which sets the default resource characteristics, presents problems for the development of a truly general cross-linguistic framework that is applicable to highly dissimilar languages. Furthermore, the complexity of syntactic analyses that require extensive use of structural modalities tends to encourage the selection of default logics that are stronger than might otherwise be chosen, with a concomitant loss of resource sensitivity that might prove valuable.

It is this earlier work using structural modalities that provides the inspiration for the hybrid approach, for which the aim is to eliminate the need of structural modalities, whilst maintaining the descriptive power they provide. The specific idea for how this may be achieved is suggested by *embedding translations*, which are methods by which one logic may be embedded within another.

3 Structural modalities and embedding translations

Let us consider how structural modalities may be used to allow controlled involvement of structural rules which are otherwise unavailable in a system. A structural modality may be associated with a modified structural rule that requires that one of the formulas directly affected by its use is marked with a given modality. For example, in a system where permutation is not freely available, *restricted* involvement of [P] might be allowed via a unary operator $\triangle$, having the sequent rules (1) (where $\triangle\Gamma$ indicates a configuration in which all types have the form $\triangle$X):

(1) $$\frac{\Gamma[(\triangle B,\ C)] \Rightarrow A}{\Gamma[(C,\ \triangle B)] \Rightarrow A}[\triangle P] \qquad \frac{\triangle\Gamma \Rightarrow A}{\triangle\Gamma \Rightarrow \triangle A}[\triangle R] \qquad \frac{\Gamma[B] \Rightarrow A}{\Gamma[\triangle B] \Rightarrow A}[\triangle L]$$

The restricted permutation rule [$\triangle$P] allows any formula of the form $\triangle$X to permute freely, i.e. undermining linear order for just this assumption. The other rules are as for necessity in the modal logic S4. The left rule [$\triangle$L] plays a key role in allowing a $\triangle$ modality to be discarded (e.g. we have the theorem $\triangle X \Rightarrow X$), so that a modal permutable assumption can also behave as an ordinary non-modal one having a specific linear position. Such permutative modalities have been used in treatments of extraction, in particular being needed to allow for the fact that moved elements do not only originate in left or right peripheral position within the domains from which they are extracted. Other structural modalities can likewise be used to give controlled reintroduction of other structural rules, e.g. an associativity modality could be used within a non-associative system. The *exponentials* ! and ? of linear logic allow controlled reintroduction of the Contraction and Weakening structural rules.

[3]The original structural modalities are linear logic's 'exponentials'. That structural modalities be used for linguistic purposes is proposed by Morrill *et al.* [7] (see also [1]), who suggest a number of modalities together with possible linguistic uses.

Structural modalities allow that stronger logics may be embedded within weaker ones, via *embedding translations.* For example, the translation (2) embeds a fragment of **LP** within **L**, so that $\Gamma \Rightarrow A$ is a theorem of the former *iff* $\triangle|\Gamma| \Rightarrow |A|$ is a theorem of the latter (see [2] for discussion). Another example is that intuitionistic logic can be embedded within (intuitionistic) linear logic, via an embedding translation using the exponentials ! and ?. Such an embedding shows, of course, that when the weaker logic is augmented with the appropriate structural modalities, the resulting system is anything but 'weaker' than the stronger logic it can embed.

(2)
$$
\begin{array}{lll}
|A| & := & A \quad \text{(A atomic)} \\
|(A \otimes B)| & := & ((\triangle|A|) \bullet (\triangle|B|)) \\
|(B \multimap A)| & := & ((\triangle|B|) \setminus |A|) \\
|(A \circ\!\!- B)| & := & (|A| \,/\, (\triangle|B|))
\end{array}
$$

4 Relations between substructural levels

Consider again the embedding translation (2). The embedding shows that **LP** can be 'represented' within the system '**L** *plus* permutation modality' ('**L**△'). Of course, it is trivially true that **L** can also be 'represented' within **L**△, and so **L**△ provides a realm in which we can, in a sense, observe coexistence of **L** and **LP** (or at least coexistence of 'images' of these systems), and observe how the two systems interrelate.

Consider, for example, the **LP** formula $X \otimes Y$ and its translation $(\triangle X) \bullet (\triangle Y)$ under (2) (or strictly its translation assuming X,Y are atomic). The modalities indicate that the X,Y subformulas may appear in either order, i.e. we observe the interderivability $(\triangle X) \bullet (\triangle Y) \Leftrightarrow (\triangle Y) \bullet (\triangle X)$, akin to $X \otimes Y \Leftrightarrow Y \otimes X$ for the original formula. The △ modalities may be 'discarded', e.g. we have $(\triangle X) \bullet (\triangle Y) \Rightarrow X \bullet Y$ (and also $(\triangle X) \bullet (\triangle Y) \Rightarrow Y \bullet X$), a step corresponding to selection of, or commitment to, one of the two permitted orders. This latter theorem suggests how $\otimes$ and $\bullet$ might be related in a multimodal logic, i.e. so that $X \otimes Y \Rightarrow X \bullet Y$ is a theorem, as if $X \otimes Y$ is in some sense 'implicitly modalised' relative to $X \bullet Y$.

Consider next the **LP** implicational $X \circ\!\!- Y$, and its translation $X/(\triangle Y)$. This formula exhibits the interderivability $X/(\triangle Y) \Leftrightarrow (\triangle Y)\backslash X$, akin to $X \circ\!\!- Y \Leftrightarrow Y \multimap X$. Note that **L**△ allows the transition $X/Y \Rightarrow X/(\triangle Y)$, suggesting $X/Y \Rightarrow X \circ\!\!- Y$ as a theorem of a mixed logic where there is coexistence of these two levels. (Note how product and residuals exhibit complementary direction of movement between levels in such characteristic inter-level transformations.)

The above discussion of embedding translations suggests a direction of linkage between **L** and **LP** that precisely contradicts the M&O view. The same line of thought suggests linkages for other levels yielding characteristic theorems such as $X \odot Y \Rightarrow X \bullet Y$, and $X \odot Y \Rightarrow X \otimes Y$, again precisely contradicting what is expected under the M&O view.

5 Formulating a hybrid system

The different views of how substructural levels should be related in a multimodal system do not require any differences to how the logic is formulated. The crucial practical difference, as might be expected, is a simple reversal of the statement of inclusion structural rules. As regards interpretation, the multimodal frame semantics presented in the main paper can equally well be used with the hybrid direction of linkage.[4]

The hybrid direction of linkage, however, *does* have some consequences for how a hybrid system may be used for linguistic analysis, specifically in regard to the treatment of word order. In particular, we cannot handle the word order consequences of linguistic derivations via the ordering of formulas within antecedent configurations. This it because it is a characteristic of any hybrid

[4] Natasha Kurtonina has pointed out to me that problems may arise in the semantics for the hybrid direction of linkage should the levels available go beyond the purely intuitionistic to include also classical. Note that Hepple [3] uses a variant of a groupoid semantics, rather than a ternary frame semantics.

system which includes **LP** as a level that, for any theorem, there exist also alternative theorems for combining the same antecedent types under all alternative orderings.[5]

One possible response to this problem might be to limit our attention, for linguistic purposes, to only those sequents whose antecedents are configured with structural connectives that *do* clearly order their subconfigurations, e.g. so that $(\cdot,\cdot)^{\bullet}$ may appear in the end-sequent of a linguistic derivation, but not $(\cdot,\cdot)^{\otimes}$, although the latter might appear elsewhere in the body of the proof.[6] Such a requirement would seem too strict, however, completely ruling out the possibility of having lexical type assignments that do not strictly order functor and argument, a decision which should surely rather be in the hands of the grammar writer.

The solution to this problem pursued in Hepple [3] is based on a system of term assignment, with the word order consequences of proofs being derived from their proof terms. The method requires a rich term algebra, together with an appropriate labelling discipline, so that proof terms fully record the natural deduction structure of the sequent proofs that give rise to them. Hence, the algebra must include, for example, a different abstraction and application operator for each different implicational operator (being needed to label its Left and Right inferences, respectively), and so on. In deriving the word order consequences of a proof, its proof term is first normalised. Normalisation simplifies a term to a form in which the argument relations amongst its free variables (which identify the antecedent formulas of the proof) are most directly expressed, by operators that encode the linear order, etc, information that was originally specified in the connectives of the types combined. This information can then be used in deriving an ordering over the free variables of the term, which in turn implies an ordering of the lexical types combined. I will not provide a detailed specification of the method here.

6 The hybrid linguistic model

I noted earlier that one problem of a structural modality based approach is that it tends to favour the selection of a relatively strong logic for the 'default' level, i.e. because the weaker the base level logic, the more extensive will be the need to use structural modalities, and so the more complex will be the overall analysis. The choice of a stronger base logic will be associated with a loss of potentially useful resource sensitivity. This problem does not arise for the hybrid approach, which freely allows us to use weaker logics in specifying lexical types that richly encode linguistic information.

For example, consider a multimodal system that includes only the levels **L** and **LP**. Of these two levels, **L** is clearly the one that will in general be more appropriate for linguistic description. Under the hybrid view, the linkage between these two levels is such that $X{\otimes}Y \Rightarrow X{\bullet}Y$ is a theorem, alongside which we will find also (e.g.) $X/Y \Rightarrow X{\circ\!\!-}Y$. Note that it is the latter theorem, and its variants, that most crucially bear upon what is gained by the move to a mixed system, given that the lexical encoding of linguistic information predominantly involves the assignment of *functional* types. Hence, a lexical functor constructed with **L** connectives may be transformed to one involving **LP** connectives, allowing us to exploit the structural freedom of that level. For example, the availability of the **LP** level, and its permutative character, allows for a possible treatment of extraction phenomena, whereby a 'sentence missing NP somewhere' may be derived as $s{\circ\!\!-}np$, as for example in the ordered theorem (3a) (where np types have been indexed to aid readability). Note that with the converse direction of linkage, but with lexical functors still constructed using **L** connectives, no practical use could be made of the permutative level in this minimal mixed system.

(3) a. $np_1,\ ((np_1\backslash s)/pp)/np_2,\ pp \Rightarrow s{\circ\!\!-}np_2$

b. $np,\ (np \mathbin{\backslash\!\!\!\circ} s)\mathbin{/\!\!\!\circ} pp \Rightarrow s/pp$

[5] Any proof of $\Gamma \Rightarrow A$ may be extended by multiple $[\sqsubseteq]$ inferences to give a proof of $\Gamma' \Rightarrow A$, where Γ' is just like Γ except all bracket pairs are $()^{\otimes}$. Extending this proof with repeated uses of [P] and [A], we can attain any desired reordering of the component types.

[6] This idea is suggested by the Moortgat & Oehrle [6] treatment of head wrapping phenomena, where certain structural connectives are designated as 'abstract', meaning 'not phonetically interpretable', and hence are not allowed to appear in sequents corresponding to linguistic combinations.

Consider next a multimodal system that includes also the level **NL**. This additional level might be adopted as the principal level for lexical specification, giving various advantages for linguistic analysis. For example, by having a lexical element subcategorise for a complement that is some 'non-associative functor' (i.e. of the form $A\oslash B$ or $B\obslash A$), we could be sure that the complement taken was a 'natural projection' of some lexical head, and not one built by composition (or other associativity based combination). On the other hand, where the freedom of associative combination is required, it is still available, given that we have (e.g.) $X\oslash Y \Rightarrow X/Y$. For example, a non-constituent coordination example such as *Mary spoke and Susan whispered, to Bill* could be derived in line with a familiar categorial analysis based on flexible structure assignment, with the conjuncts being coordinated under type s/np, since we have the theorem (3b). Furthermore, since we have also $X\oslash Y \Rightarrow X\circ\!\!-Y$, such non-associative lexical specification is still compatible with the treatment of extraction hinted at above.

It is hoped that the above simple examples of linguistic uses of multimodal systems will serve to give a feeling for the general character of the linguistic model that the hybrid approach would favour, i.e. one with very rich lexical encoding of syntactic information, achieved using predominantly the connectives of the *weakest* available logic, with the stronger logics of the mixed system allowing less informative (but hence also more 'flexible') descriptions of linguistic objects. The above example systems clearly do not exhaust the possibilities for 'rich lexical encoding'. It seems likely that lexical assignments should specify *headedness* or *dependency* information, as in the calculi of Moortgat & Morrill [4], specifying for a function and its argument, for example, which of the two is head. Further distinctions along this line might be encoded. For example, we might make a distinction between head-dependent and head-adjunct relations, and even different kinds of head-dependent relation (e.g. 'internal' versus 'external' argument), and so on. Speaking loosely, the grammar writer can feel free to include such distinctions in lexical specifications, in the knowledge that they can readily be ignored where they are not required.

7 Discussion

Let us next look at the two opposing views side by side, and attempt to cast some light upon their difference of opinion. Despite their diametrically opposing claims for what are natural linkages, the two views have both been defended in terms of giving inter-level transformations that involve 'forgetting' or 'loss of information'. Thus, for the M&O view, a transformation such as $X\bullet Y \Rightarrow X\otimes Y$ involves the forgetting of order, since **LP** is permutative. In general for this view, the more structural rules that a level has, the 'less informative' it is seen to be, since the less the structure that is preserved at that level. In the hybrid view, the permutativity $X\otimes Y \Leftrightarrow Y\otimes X$ is taken to indicate that *both* orders are possible, rather than that the ordering is unknown, and so the transformation $X\otimes Y \Rightarrow X\bullet Y$ can again be seen as forgetting, i.e. forgetting of one the two orders that are possible. In general for the hybrid view, the more structural rules a level has, the *more* informative it is seen to be, since the structural rules are seen to indicate a conjunction of alternative possibilities.

One point of comparison for the two views is how they each contribute to actual linguistic accounts. I have briefly indicated some linguistic uses for the hybrid view of linkage above, which are detailed, along with some further uses, in Hepple [3]. For linguistic uses of the M&O view, we must look to Moortgat & Oehrle [6], where their approach is used in handling head wrapping phenomena. For example, an account of verb-raising in Dutch and German is provided, for which much of the work is done by interaction rules, which operate to give local reordering of elements, and thereby, through recursion, implement infixation. The role for inclusion rules (i.e. the linkage cases that are at issue) is in some sense small, but still important, being needed to provide a concatenative 'base case' for the recursively effected infixation process. An example of one such inclusion (shown as a deductive formulation axiom) is: $A \circ_l B \longrightarrow A \circ_{lh} B$

The product $\circ_l$ indicates that its left subtype determines the head of the construction, and carries concatenative import concerning word order. The product $\circ_{lh}$ indicates that its components combine via infixation, where it is the left element that infixes and that also determines the head of the construction. It is not at all obvious that this axiom accords with the M&O idea that inclusions arise through forgetting of structure, i.e. that an constructor indicating infixation is 'less informative' than one which carries concatenative import, rather than that the two are simply incomparable. If, as suggested above, we take the structural rules that are present at a particular level as an indicator of its informativeness, there is again no indication that one level should include the other, i.e. both are without level-internal structural rules. All structural rules relating to these constructors concern *interaction* of levels.

One possibility that we should address is that the two opposing views of linkage are *both wrong*, i.e. wrong in claiming that certain patterns of linkage *must* obtain (because some links are more 'natural' than others). In that case, the two opposing views may also be *both right*, i.e. right in allowing the linkages that they do allow. In particular, it is possible that the debate as to which view is correct is flawed by the lack of clear enough understanding of how to identify the 'meaning' of any one level, as a basis for determining appropriate linkages. It may be that the 'meaning' of a level cannot be determined purely from level-internal considerations, because a vital component of this meaning is the level's linkages to other levels and its place within an overall multimodal system. For example, the hybrid view claim that the interderivability $X \otimes Y \Leftrightarrow Y \otimes X$ indicates that both orders are possible would have little content were it not for the linkage to the ordered $\bullet$-level, which allows the possible orders to be 'cashed out'. On the other hand, for the M&O view, the claim that the transformation $X \bullet Y \Rightarrow X \otimes Y$ involves the loss of order information crucially depends on the fact that there is no further transformation possible to another level at which order is preserved (indeed the availability of such a transformation would change not only the meaning of the $\otimes$-level, but also of the 'prior' $\bullet$-level). If this idea is correct, the $\otimes$-levels of the two opposing views are in fact non-equatable, having different meanings due to their different linkages. It may even be that two such subsystems may *both* appear within a single multimodal system (given obvious notational separation).

Perhaps then, the crucial requirement in constructing a multimodal system is not that its component levels *must* be ordered in some fixed manner, according to some criterion based on level-internal characteristics, but rather that derivability within the system as a whole is well-behaved regarding "loss of information" in inter-level transitions.

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