Lemma

If a PDA recognises a language, then it is context free

take PDA from start state to accept state, with empty stack

Proof Sketch (by Construction)

For PDA $P = (Q,\Sigma,\Gamma,\delta,q_0,\{q_{accept}\})$ construct G with variables $\{A_{pq} \mid p,q \in Q\}$, start variable $A_{q_0,q_{accept}}$ and rules:

1. $A_{pq} \to aA_{rs}b$

   for each $p,q,r,s \in Q$, $t \in \Gamma$ and $a,b \in \Sigma_\varepsilon$, if $\delta(p,a,\varepsilon)$ contains (r,t) and $\delta(s,b,t)$ contains (q,ε)

   Q Γ $\neq \varepsilon$ Q Σ_ε Γ_ε

2. $A_{pq} \to A_{pr}A_{rq}$

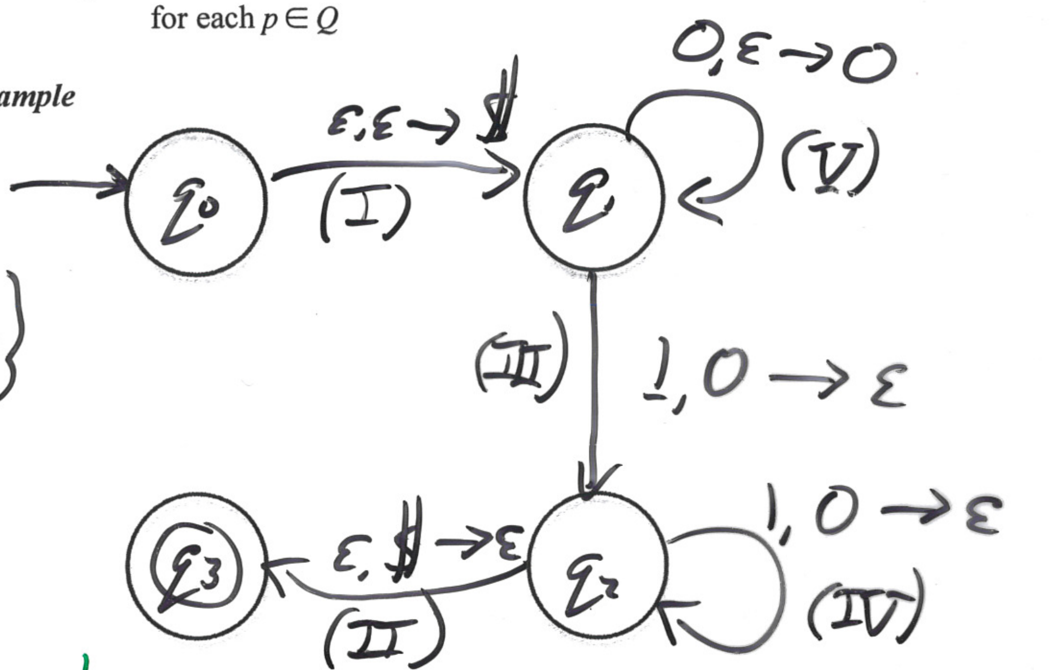
   for each $p,q,r \in Q$

3. $A_{pp} \to \varepsilon$

   for each $p \in Q$

Example

$\{0^n1^n \mid n>0\}$



p a r t

(I) $\delta(q_0,\varepsilon,\varepsilon) = \{(q_1,\$),\ldots\}$

(II) $\delta(q_2,\varepsilon,\$) = \{(q_3,\varepsilon),\ldots\}$

s b t q

(III) $\delta(q_1,1,0) = \{(q_2,\varepsilon),\ldots\}$

(V) $\delta(q_1,0,\varepsilon) = \{(q_1,0),\ldots\}$

(IV) $\delta(q_2,1,0) = \{(q_2,\varepsilon),\ldots\}$

1. 1. 1.

$A_{q_0,q_3} \to \varepsilon A_{q_1q_2} \varepsilon$

$A_{q_1q_2} \to 0 A_{q_1q_2} 1$

$A_{q_1q_2} \to 0 A_{q_1q_1} 1$

$A_{q_2q_2} \to \varepsilon$

$A_{q_1q_1} \to \varepsilon$ (by rule 3)

Claim (proof in Sipser)

A_{pq} generates string $x \Leftrightarrow x$ can bring PDA from state p with empty stack to q with empty stack