

Turing Machine Variants, Equivalence, Decidability – Lecture 14
James Marshall

Levels of Description for Turing-Machines

- *formal*
- *implementation*
- *high-level*

Definition

A Multitape Turing Machine is a Turing Machine with $k > 1$ tapes, each with its own read-write head. The transition function then has the form

$$\delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R, S\}^k$$

Theorem

Every Multitape Turing Machine has an equivalent single-tape Turing Machine

Proof Sketch (by Construction)

Definition

A Nondeterministic Turing Machine is a Turing Machine that can have more than one valid transition for any state-read pair, so the transition function has the form

$$\delta: Q \times \Gamma^k \rightarrow P(Q \times \Gamma \times \{L, R\})$$

Theorem

Every Nondeterministic Turing Machine has an equivalent deterministic Turing Machine

Proof Sketch (by Construction)

Decidability Examples

$$A_{DFA} = \{\langle B, w \rangle \mid B \text{ is a DFA that accepts } w\}$$

$$E_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \emptyset\}$$

$$EQ_{DFA} = \{\langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$$

Context-free Languages

Language Hierarchy