Regular Expressions - Lecture 5

James Marshall

Definition (inductive) - Regular Expression

R is a ***regular expression*** if R is

1. a for any a in the alphabet Σ
2. ε
3. $\varnothing$
4. $(A \cup B)$ where A and B are regular expressions
5. $(A \circ B)$ where A and B are regular expressions
6. (A^*)

Applications of regular expressions

Some further notation and some identities

Examples - Give regular expressions in the alphabet $\Sigma = \{0, 1\}$ for

$\{w \mid w$ contains the substring 010 within it$\}$

$\{w \mid w$ may or may not start with a 0, followed by any number of 1s$\}$

$\{w \mid w$ begins with 0 and ends with 1, or begins with 1 and ends with 0, such that no two 0s or 1s are ever adjacent$\}$

Theorem

A language is regular iff (if and only if) some regular expression describes it

(language A is described by a regular expression $\Leftrightarrow$ language A is regular)

Lemma

language A is described by a regular expression $\Rightarrow$ language A is regular

Proof (by construction)

1. $L(R) = \{a\}$

2. $L(R) = \{\varepsilon\}$

3. $L(R) = \varnothing$

4. $L(R) = L(A \cup B)$

5. $L(R) = L(A \circ B)$

6. $L(R) = L(A^*)$