Nonregular Languages - Lecture 7

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Examples of nonregular languages (?):

$C = \{w \mid w$ has an equal number of 0s and 1s$\}$

$D = \{ww^{\mathcal{R}}\}$

An examples that looks nonregular but isn't:

$E = \{w \mid w$ has an equal number of substrings 01 and 10$\}$

Exercise: prove the regularity of E

Theorem - ***The Pumping Lemma for Regular Languages***

If A is a regular language, then there is a minimum length p (the ***pumping length***) where, for any string $s \in A$ of length at least p, s can be divided into three pieces $s = xyz$ such that

1. $xy^iz \in A$, for each $i \geq 0$
2. $|y| > 0$
3. $|xy| \leq p$

Proof Sketch

1. Consider a machine M that recognises A

2. Choose a string $s \in A$ of length $n \geq p$, where p = the number of states in M

3. Let $q_1q_2...q_{n+1}$ be the sequence of states M goes through in accepting s

4. By the ***pigeonhole principle***, since $n \geq p$, one or more states in M must be visited more than once

5. Consider the first $p + 1$ states visited by M; one state (labelled q_r) must be visited twice, so s can be broken down as follows:

Example 1:

Prove that $F = \{0^n1^n \mid n \geq 0\}$ is not regular, by application of the pumping lemma

Example 2:

Prove that $G = \{w \mid w$ has an equal number of 0s and 1s$\}$ is not regular, *without* recourse to the pumping lemma