Context Free Grammars - Lecture 8

James Marshall

Recall that not all languages are regular (as demonstrated for particular examples by application of the *pumping lemma*). For example (from the previous lecture):

$$F = \{0^n 1^n \mid n \geq 0\}$$

The above example *can* be described, however, using a **context-free grammar**, as follows:

$$A \rightarrow 0A1$$
$$A \rightarrow B$$
$$B \rightarrow \varepsilon$$

(or, equivalently, $A \rightarrow 0A1 \mid \varepsilon$). Thus F is a **context-free language**.

Parse-tree for $000111 \in F$:

Definition - Context Free Grammar (CFG)

A ***context-free grammar*** *is a 4-tuple* (V, Σ, R, S) where

1. *V is the finite set of* ***variables***

2. *Σ is a finite set of* ***terminals****, where* $\Sigma \cap V = \emptyset$

3. *R is a set of* ***rules*** *of form* $A \rightarrow w$ where $A \in V$ and w is a string of variables and terminals

4. *$S \in V$ is the* ***start variable***

Definition - The Language of a Grammar

For a grammar $G = (V, \Sigma, R, S)$, if u, v, and w are strings of variables ($\in V$) and terminals ($\in \Sigma$), and $A \rightarrow w$ is a rule in the grammar G (*i.e.* a member of R), then uAv **yields** uwv. This is written $uAv \Rightarrow uwv$. Then u **derives** v ($u \Rightarrow^* v$) if $u = v$ or a sequence of the form $u_1, u_2, ..., u_k$ exists for $k \geq 0$ such that

$$u_1 \Rightarrow u_2 \Rightarrow ... \Rightarrow u_k \Rightarrow v$$

Then $L(G) = \{w \in \Sigma^* \mid S \Rightarrow^* w\}$

Example:

<EXPR> → <EXPR>+<TERM> | <TERM>
<TERM> → <TERM>×<FACTOR> | <FACTOR>
<FACTOR> → (<EXPR>) | a

Exercise: draw a parse tree for a×a

Exercise: draw a parse tree for (a+a)×(a+a)

Applications:

Language hierarchy: